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“IGOR SIKORSKY KYIV POLYTECHNIC INSTITUTE”

PHYSICS:

MECHANICS. MOLECULAR PHYSICS.

THERMODYNAMICS:

Practical trainings

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Compiler:

Chursanova Maryna Valeriivna – docent, PhD

Editor-in-Chief:

Gorshkov V.M. – director of the Department of General Physics and Solid State Physics, Doctor of Physical and Mathematical Sciences, Professor

Reviewer:

Reshetnyak S.O. – professor of the Department of General and Experimental Physics, Doctor of Physical and Mathematical Sciences, Professor.

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FUNDAMENTALS OF MECHANICS

Topic 1. Kinematics of translational and rotational motion.

General problem-solving strategy:

1. Carefully read the text of the problem. Study any representations of the information (for example, diagrams, graphs, tables) that accompany the problem. Analize the given data and recall what physical laws they are subject to. Simplify the problem. Remove the details that are not important to the solution. For example, model a moving object as a particle. If appropriate, ignore air resistance or friction between a sliding object and a surface.

2. Make a short record of the data. Write in the column on the left all the values given in the task and the values you need to find. It is better to express all the numerical data in the Universal system of units (SI).

3. If a pictorial representation is not provided, make a quick drawing of the situation. Indicate any known values directly on your sketch.

4. Write the general form of an equation or a system of equations that represent the physical process described in the problem.

5. If equations are in the vector form, represent them in the scalar form by finding projections onto the coordinate axes *Ox*, *Oy* and *Oz*.

6. Analyzing conditions of the problem, express the general equations only in terms of the physical values that are mentioned in the problem or can be taken from the tables of physical constants.

7. Use algebra (and calculus, if necessary) to solve the equations symbolically for the unknown variable in terms of what is given. Obtain the "working formula".

8. Make calculations. Substitute in the appropriate numbers, calculate the result, and round it to the proper number of significant figures.

9. Make verification of the units, substituting them into the "working formula". The resulting unit must coincide with the units of the found physical value.

Example 1.1 Definitions of the kinematic variables.

Position vector of a particle is changing with time according to the equation $\vec{r} = 3t^2\vec{i} + 2t\vec{j} + \vec{k}$. Determine velocity, acceleration and magnitudes of the velocity and acceleration at the moment t = 1 s. What is the path covered by the particle during the 11-th second of motion?

Solution:

According to the definition, velocity vector equals to the time derivative of the position vector:

$$\vec{v} = \frac{d\vec{r}}{dt} = 6t\vec{i} + 2\vec{j}.$$

The speed

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(6t)^2 + (2)^2} = \sqrt{36t^2 + 4}.$$

Acceleration vector is the time derivative of the velocity vector:

$$\vec{a} = \frac{d\vec{v}}{dt} = 6\vec{i}.$$

Its magnitude a = 6.

At the moment t = 1 s

$\vec{v} = 6\vec{i} + 2\vec{j}$ (m/s); $v = 2\sqrt{10}$ (m/s); $\vec{a} = 6\vec{i}$ (m/s^2); a = 6 (m/s^2).

The path covered by the particle during the 11-th second of motion is equal to the magnitude of the displacement vector between the 11-th and the 10-th seconds of motion:

$$s = |\Delta\vec{r}| = |\vec{r}(11) - \vec{r}(10)| = |3(11^2 - 10^2)\vec{i} + 2(11-10)\vec{j}| = |63\vec{i} + 2\vec{j}| = \sqrt{63^2 + 4} \approx 63\,\text{(m)}.$$

Example 1.2 Projectile motion.

Two bodies are thrown horizontally from the same point but in the opposite directions with initial speeds v_{01} = 3 m/s and v_{02} = 4 m/s. Find the distance between the bodies at the moment when their velocity vectors become perpendicular to each other.

Solution:

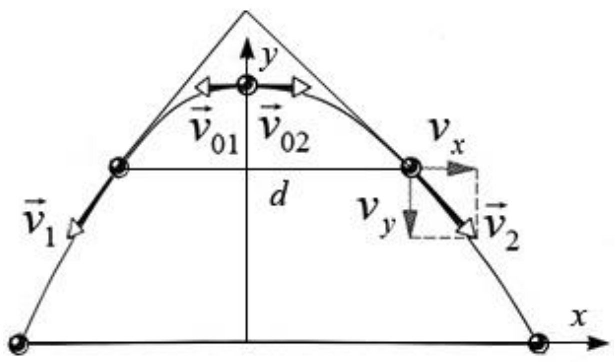


Figure 1.1.

When to vectors are perpendicular to each other their dot product equals zero. So,

$$\vec{v}_1 \cdot \vec{v}_2 = 0,$$

where $\vec{v}_1$ and $\vec{v}_2$ are the velocity vectors of the first and second body, respectively.

As we know,

$$\vec{v}_1 \cdot \vec{v}_2 = v_{1x} v_{2x} + v_{1y} v_{2y}.$$

We need to find components of the velocity for the two bodies. Free fall of the object is the case of the uniformly accelerated motion with the acceleration due to gravity $\vec{a} = \vec{g}$ (the projectile motion). The kinematic equation for the velocity in such case is given by the formula

$$\vec{v} = \vec{v}_0 + \vec{g}t.$$

where $\vec{v}_0$ is the initial velocity of the body.

Let's find projections of this equations onto the coordinate axes *Ox* and *Oy* for the two bodies (see Figure 1.1.):

$$\begin{cases} v_{1x} = -v_{01} \\ v_{1y} = -gt \end{cases}; \qquad \begin{cases} v_{2x} = v_{02} \\ v_{2y} = -gt \end{cases}.$$

Substituting these values into the expression for the dot product, we obtain:

$$-v_{01} \cdot v_{02} + (gt)^2 = 0,$$

and the moment of time when the velocity vectors become perpendicular to each other is

$$t = \frac{\sqrt{v_{01} \cdot v_{02}}}{g}.$$

Now we can find the distance between the two bodies using the formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2},$$

where x_1, y_1, x_2, y_2 are coordinates of the first and second bodies.

The kinematic equation for the position vector in the case of uniformly accelerated motion is given by the formula

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{\vec{g}t^2}{2},$$

and its projections onto the coordinate axes *Ox* and *Oy* for the two bodies are:

$$\begin{cases} x_1 = -v_{01}t \\ y_1 = h - \dfrac{gt^2}{2} \end{cases}; \qquad \begin{cases} x_2 = v_{02}t \\ y_2 = h - \dfrac{gt^2}{2} \end{cases}.$$

As we see, $y_1 = y_2$, then

$$d = \sqrt{(x_1 - x_2)^2} = |x_1 - x_2| = |-v_{01}t - v_{02}t| = (v_{01} + v_{02})t,$$

and substituting the fond expression for t we obtain:

$$d = (v_{01} + v_{02})\frac{\sqrt{v_{01} \cdot v_{02}}}{g}.$$

Calculations:

$$d = (3 + 4)\frac{\sqrt{3 \cdot 4}}{9.8} \approx 2.47 \text{ (m)}.$$

Example 1.3 Untypical curvilinear motion

An air balloon starts to rise vertically from the ground at constant velocity of magnitude v_0. It acquires horizontal component of the velocity due to the wind, $v_x = \alpha y$, where α is a coefficient, y is an elevation of the balloon above the ground. Find the drift of the balloon $x(y)$ and its acceleration.

Solution:

As given, components of the velocity of the air balloon are:

$$\begin{cases} v_x = \alpha y \\ v_y = v_0 \end{cases}.$$

We know that $v_x = \dfrac{dx}{dt}$ and $v_y = \dfrac{dy}{dt}$. Let's transform the equations for the velocity components to express them only in terms of the coordinates x and y.

$$\frac{dx}{dt} = \alpha y; \qquad \frac{dx}{dt} \cdot \frac{dy}{dy} = \alpha y; \qquad \frac{dx}{dy} \cdot \frac{dy}{dt} = \alpha y,$$

but $\frac{dy}{dt} = v_y = v_0$, and we obtain

$$v_0 \cdot \frac{dx}{dy} = \alpha y.$$

Now we can separate variables and take integral of the both sides of the equation considering the fact that the balloon starts to rise from the ground, that is its initial coordinates equal zero:

$$v_0 dx = \alpha y dy; \qquad \int_0^x v_0 dx = \int_0^y \alpha y dy; \qquad v_0 x = \frac{\alpha y^2}{2}.$$

So, the drift of the balloon

$$x = \frac{\alpha}{2v_0} y^2.$$

Now let’s find the acceleration of the balloon. Its components

$$\begin{cases} a_x = \dfrac{dv_x}{dt} = \dfrac{d(\alpha y)}{dt} \\ a_y = \dfrac{dv_y}{dt} = \dfrac{dv_0}{dt} \end{cases}.$$

We see that the vertical component $a_y = 0$ because v_0 is constant. The horizontal component is

$$a_x = \alpha \frac{dy}{dt} = \alpha \cdot v_y = \alpha \cdot v_0.$$

So the total acceleration of the balloon is directed horizontally and equal to

$$a = \alpha \cdot v_0.$$

Example 1.4 Total, tangential and radial acceleration.

A stone is thrown horizontally with the initial speed v_0 = 30 m/s from the certain height. Define its speed, tangential and radial acceleration at the moment t = 2 s from the beginning of the motion.

Solution:

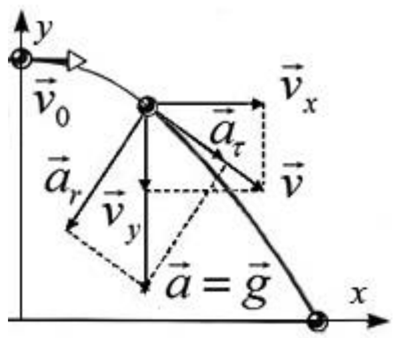


Figure 1.2.

This is the case of the projectile motion with the free-fall acceleration. The kinematic equation for the velocity of the stone is given by the formula

$$\vec{v} = \vec{v}_0 + \vec{g}t .$$

Components of the velocity are (see Figure 1.2.)

$$\begin{cases} v_x = v_0 \\ v_y = -gt \end{cases} .$$

Then the speed of the stone can be found as

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + g^2t^2} .$$

By definition, the tangential acceleration of the body is due to the changing magnitude of the velocity (changing speed), that is

$$a_\tau = \frac{dv}{dt} = \frac{d}{dt}\left(\sqrt{v_0^2 + g^2t^2}\right) = \frac{1}{2\sqrt{v_0^2 + g^2t^2}} \cdot 2g^2t .$$

So, $a_\tau = \dfrac{g^2t}{\sqrt{v_0^2 + g^2t^2}}$.

The radial acceleration is due to the change in direction of the velocity, but the radius of the curvature of the trajectory is unknown. However, we know the total acceleration of the stone, which is equal to the free-fall acceleration g. Then,

$$g = \sqrt{a_\tau^2 + a_r^2} ;$$

$$a_r = \sqrt{g^2 - a_\tau^2} = \sqrt{g^2 - \frac{g^4t^2}{v_0^2 + g^2t^2}} = \frac{v_0 g}{\sqrt{v_0^2 + g^2t^2}} .$$

Calculations:

$$v = \sqrt{30^2 + 9.8^2 \cdot 2^2} \approx 35.8 \text{ (m/s)}$$

$$a_\tau = \frac{9.8^2 \cdot 2}{\sqrt{30^2 + 9.8^2 \cdot 2^2}} = 5.37 \text{ (m/s}^2\text{)}$$

$$a_r = \frac{30 \cdot 9.8}{\sqrt{30^2 + 9.8^2 \cdot 2^2}} \approx 8.21 \text{ (m/s}^2\text{)}$$

Example 1.5 Circular motion

A disk of radius R = 20 cm is rotating according to the equation φ = A + Bt + Ct^3, where A = 3 rad, B = -1 rad/s, C = 0.1 rad/s^3. Find the tangential, radial and total acceleration of the points on the rim of the disk at the moment t = 10 s.

Solution:

Magnitude of the tangential acceleration

$$a_\tau = \frac{dv}{dt} = R\frac{d\omega}{dt},$$

where ω is the angular speed of the point. Angular speed can be found as the time derivative of the angular position:

$$\omega = \frac{d\varphi}{dt} = \frac{d(A + Bt + Ct^3)}{dt} = B + 3Ct^2.$$

Then,

$$a_\tau = R\frac{d(B + 3Ct^2)}{dt} = 6CtR.$$

Magnitude of the radial acceleration

$$a_r = \frac{v^2}{R} = \omega^2 R = \left(B + 3Ct^2\right)^2 R.$$

Magnitude of the total acceleration

$$a = \sqrt{a_\tau^2 + a_r^2}.$$

Calculations:

$$a_\tau = 6\cdot 0.1\cdot 10\cdot 0.2 = 1.2\,(\text{m/s}^2)$$

$$a_r = \left(-1 + 3\cdot 0.1\cdot 10^2\right)^2 0.2 = 168.2\,(\text{m/s}^2)$$

$$a = \sqrt{1.2^2 + 168.2^2} \approx 168.2\,(\text{m/s}^2)$$

Problems

1. A person walks first at a constant speed of 5.00 m/s along a straight line from point A to point B and then back along the line from B to A at a constant speed of 3.00 m/s. (a) What is her average speed over the entire trip? (b) What is her average velocity over the entire trip?
2. A particle moves according to the equation $x = 10t^2$, where x is in meters and t is in seconds. (a) Find the average velocity for the time interval from 2.00 s to 3.00 s. (b) Find the average velocity for the time interval from 2.00 to 2.10 s.
3. A particle moves along the x axis according to the equation $x = 2.00 + 3.00t - 1.00t^2$, where x is in meters and t is in seconds. At $t = 3.00$ s, find (a) the position of the particle, (b) its velocity, and (c) its acceleration.
4. A cannon shell is fired straight up from the ground at an initial speed of 225 m/s. After how much time is the shell at a height of 620 m above the ground and moving downward?
5. An arrow is shot straight up in the air at an initial speed of 15.0 m/s. After how much time is the arrow moving downward at a speed of 8.00 m/s?
6. The driver of a car slams on the brakes when he sees a tree blocking the road. The car slows uniformly with an acceleration of - 5.60 m/s^2 for 4.20 s, making straight skid marks 62.4 m long, all the way to the tree. With what speed does the car then strike the tree?
7. A particle moves along the x axis. Its position is given by the equation $\mathbf{x} = 2 + 3t - 4t^2$, with x in meters and t in seconds. Determine (a) its position when it changes direction and (b) its velocity when it returns to the position it had at $t = 0$.
8. A rock is thrown downward from the top of a 40.0-m-tall tower with an initial speed of 12 m/s. Assuming negligible air resistance, what is the speed of the rock just before hitting the ground?
9. On another planet, a marble is released from rest at the top of a high cliff. It falls 4.00 m in the first 1 s of its motion. Through what additional distance does it fall in the next 1 s?

10. A pebble is released from rest at a certain height and falls freely, reaching an impact speed of 4 m/s at the floor. Next, the pebble is thrown down with an initial speed of 3 m/s from the same height. What is its speed at the floor?

11. A ball is thrown directly downward with an initial speed of 8.00 m/s from a height of 30.0 m. After what time interval does it strike the ground?

12. The height of a helicopter above the ground is given by $h = 3.00t^3$, where h is in meters and t is in seconds. At $t = 2.00$ s, the helicopter releases a small mailbag. How long after its release does the mailbag reach the ground?

13. A man drops a rock into a well. (a) The man hears the sound of the splash 2.40 s after he releases the rock from rest. The speed of sound in air (at the ambient temperature) is 336 m/s. How far below the top of the well is the surface of the water? (b) If the travel time for the sound is ignored, what percentage error is introduced when the depth of the well is calculated?

14. The vector position of a particle varies in time according to the expression $\vec{r} = 3\vec{i} - 6t^2\vec{j}$. Find expressions for the velocity and acceleration of the particle as a function of time.

15. The coordinates of an object moving in the xy plane vary with time according to the equations $x = -5\sin\omega t$ and $y = 5\cos\omega t$. (a) Determine the components of velocity of the object at $t = 0$. (b) Determine the components of acceleration of the object at $t = 0$. (c) Write expressions for the position vector, the velocity vector, and the acceleration vector of the object at any time $t > 0$. (d) Describe the path of the object in an xy plot.

16. A particle initially located at the origin has an acceleration of $\vec{a} = 3\vec{j}$ m/s^2 and an initial velocity of $\vec{v}_0 = 5\vec{i}$ m/s. Find (a) the vector position of the particle at any time t, (b) the velocity of the particle at any time t, (c) the coordinates of the particle at $t = 2.00$ s, and (d) the speed of the particle at $t = 2.00$ s

17. A projectile is launched on the Earth with a certain initial velocity and moves without air resistance. Another projectile is launched with the same initial velocity on the Moon, where the acceleration due to gravity is one-sixth as large. How does

the range of the projectile on the Moon compare with that of the projectile on the Earth?

18. A projectile is fired in such a way that its horizontal range is equal to three times its maximumv height. What is the angle of projection?

19. A firefighter, a distance d from a burning building, directs a stream of water from a fire hose at angle α above the horizontal. If the initial speed of the stream is v_0, at what height h does the water strike the building?

20. A landscape architect is planning an artificial waterfall in a city park. Water flowing at 1.70 m/s will leave the end of a horizontal channel at the top of a vertical wall h = 2.35 m high, and from there it will fall into a pool. Will the space behind the waterfall be wide enough for a pedestrian walkway?

21. A soccer player kicks a rock horizontally off a 40.0-m-high cliff into a pool of water. If the player hears the sound of the splash 3.00 s later, what was the initial speed given to the rock? Assume the speed of sound in air is 343 m/s.

22. A car travels due east with a speed of 50.0 km/h. Raindrops are falling at a constant speed vertically with respect to the Earth. The traces of the rain on the side windows of the car make an angle of 60.0° with the vertical. Find the velocity of the rain with respect to (a) the car and (b) the Earth.

23. Lisa in her Lamborghini accelerates at the rate of $(3\vec{i} - 2\vec{j})$ m/s^2, while Jill in her Jaguar accelerates at $(\vec{i} + 3\vec{j})$ m/s^2. They both start from rest at the origin of an xy coordinate system. After 5.00 s, (a) what is Lisa's speed with respect to Jill, (b) how far apart are they, and (c) what is Lisa's acceleration relative to Jill?

Topic 2. Dynamics of translational motion. Forces.

General strategy for solving problems using the Newton's laws:

1. Conceptualize. Draw a simple, neat free-body diagram of the system. Establish convenient coordinate axes for each object in the system.

2. Categorize. If an acceleration component for an object is zero, the object is modeled as a particle in equilibrium in this direction and $\sum_{i=1}^{n}\vec{F}_i = 0$. If not, the object is modeled as a particle under a net force in this direction and $\sum_{i=1}^{n}\vec{F}_i = m\vec{a}$.

3. Analyze. Isolate the object whose motion is being analyzed. Draw a free-body diagram for this object. For systems containing more than one object, draw separate free-body diagrams for each object. Do not include in the free-body diagram forces exerted by the object on its surroundings.

Find the components of the forces along the coordinate axes:

$$\sum_{i=1}^{n} F_{ix} = ma_x; \qquad \sum_{i=1}^{n} F_{iy} = ma_y; \qquad \sum_{i=1}^{n} F_{iz} = ma_z.$$

Check your dimensions to make sure that all terms have units of force.

Solve the component equations for the unknowns. Remember that you generally must have as many independent equations as you have unknowns to obtain a complete solution.

4. Finalize. Make sure your results are consistent with the free-body diagram. Also check the predictions of your solutions for extreme values of the variables. By doing so, you can often detect errors in your results.

Example 2.1. A body on the inclined plane.

A block slides down an incline of angle α = 30^0 and length l = 2 m in time t = 2 s. Find the coefficient of kinetic friction between the block and the surface of the incline.

Solution:

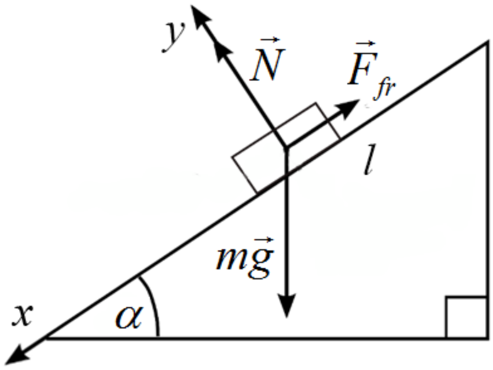


Figure 2.1.

The gravitational force, the friction force and the normal force are acting on the block. According to the Newton's second law,

$$m\vec{a} = m\vec{g} + \vec{F}_{fr} + \vec{N} .$$

Let's choose the *Ox* axis along the incline and the *Oy* axis perpendicular to it (see Figure 2.1).

Then projections of the Newton's second law onto the coordinate axes are

$$\begin{cases} Ox: ma = mg\sin\alpha - F_{fr} \\ Oy: 0 = -mg\cos\alpha + N \end{cases} .$$

We see that the normal force for the body on the incline

$$N = mg\cos\alpha .$$

Then, the friction force

$$F_{fr} = \mu N = \mu mg\cos\alpha ,$$

where μ is the coefficient of friction. After the substitution we obtain:

$$ma = mg\sin\alpha - \mu mg\cos\alpha ; \qquad a = g\sin\alpha - \mu g\cos\alpha ;$$

$$\mu = \frac{g\sin\alpha - a}{g\cos\alpha} = tg\alpha - \frac{a}{g\cos\alpha} .$$

We see that the acceleration of the block is constant and we can use the kinematic equation for the uniformly accelerated motion in order to find it:

$$x = x_0 + v_{0x}t + \frac{a_x t^2}{2} = \frac{a\,t^2}{2},$$

as the initial coordinate and the initial speed of the block equal zero and its acceleration is directed along the *Ox* axis. The length of the path equals *l*, and

$$l = \frac{a\,t^2}{2}; \qquad a = \frac{2l}{t^2}.$$

Finally,

$$\mu = tg\alpha - \frac{2l}{gt^2 \cos\alpha}.$$

Calculations:

$$\mu = tg30^0 - \frac{2 \cdot 2}{9.8 \cdot 2^2 \cos 30^0} \approx 0.46$$

Example 2.2 Body under the changing force.

A small body starts to slide down the incline of angle α. The coefficient of kinetic friction depends on the covered distance according to the relation $\mu = \gamma x$, where γ is a constant, x is the distance. Find the path covered by the body before it stops.

Solution:

The Newton's second law for the body on the incline is

$$m\vec{a} = m\vec{g} + \vec{F}_{fr} + \vec{N}.$$

Its projections onto the *Ox* and *Oy* axes (see Figure 2.1) are

$$\begin{cases} Ox: ma = mg\sin\alpha - F_{fr} \\ Oy: 0 = -mg\cos\alpha + N \end{cases}.$$

The force of friction is changing with the coordinate of the body:

$$F_{fr} = \mu N = \mu mg\cos\alpha = \gamma x mg\cos\alpha.$$

After the substitution we obtain:

$$ma = mg\sin\alpha - \gamma xmg\cos\alpha\ ;$$

$$a = g(\sin\alpha - \gamma\cos\alpha \cdot x)\,.$$

When the body stops, its speed equals zero. Acceleration and speed are related by the formula $a = \frac{dv}{dt}$. From the expression for the acceleration which we have obtained we see that it is changing with distance *x*. So let's express this equality only in terms of variables *x* and *v*:

$$a = \frac{dv}{dt} = \frac{dv}{dt} \cdot \frac{dx}{dx} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v \cdot \frac{dv}{dx}\,,$$

as $\frac{dx}{dt} = v$. Then,

$$v \cdot \frac{dv}{dx} = g(\sin\alpha - \gamma\cos\alpha \cdot x)\,; \qquad v \cdot dv = g(\sin\alpha - \gamma\cos\alpha \cdot x)dx\,.$$

After integration we obtain

$$\frac{v^2}{2} = g(\sin\alpha x - \gamma\cos\alpha \cdot \frac{x^2}{2}) = gx(\sin\alpha - \frac{\gamma}{2}\cos\alpha \cdot x)\,.$$

Now we can find that when *v* = 0,

$$\sin\alpha - \frac{\gamma}{2}\cos\alpha \cdot x = 0\,,$$

and

$$x = \frac{2}{\gamma}tg\alpha\,.$$

Example 2.3 Two objects connected by a cord

A block of mass m_1 and a block of mass m_2 are attached by a lightweight cord that passes over a frictionless pulley of negligible mass as in Figure 2.2. The block m_2 lies on a frictionless incline of angle α. Find the magnitude of the acceleration of the two objects and the tension in the cord.

Solution:

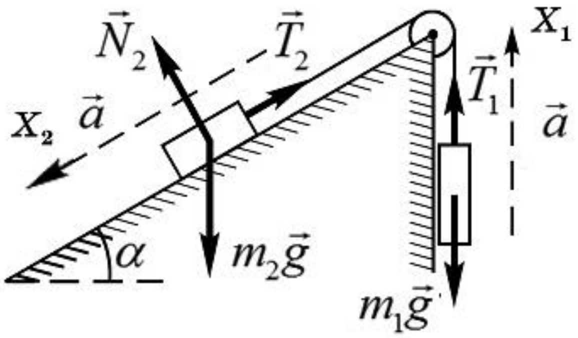


Figure 2.2.

Let's assume that m_2 moves down the incline, then m_1 moves upward. Because the objects are connected by a cord (which we assume is massless and does nor stretch), their accelerations have the same magnitude.

Consider the free-body diagrams for the bodies (Figure 2.2). The force of friction is absent because the incline is frictionless. Now we should write the Newton's second law for each of the bodies separately:

$$m_1\vec{g} + \vec{T}_1 = m_1\vec{a}\,;$$

$$m_2\vec{g} + \vec{T}_2 + \vec{N} = m_2\vec{a}\,.$$

The two bodies are interacting with each other through the cord, so we should apply the Newton's third law for the tension in the cord:

$$T_1 = T_2 = T.$$

Now let's write Newton's second law in component form.

For the first body let's choose the upward direction as positive:

$$-m_1 g + T = m_1 a\,.$$

For the second body let's choose the Ox axis along the incline:

$$m_2 a = m_2 g \sin\alpha - T \ .$$

Now we can express the tension $T = m_1(a+g)$ from the first equation and substitute it into the second equation:

$$m_2 a = m_2 g \sin\alpha - m_1(a+g) \, .$$

Solve for a:

$$a = \frac{m_2 \sin\alpha - m_1}{m_1 + m_2} \cdot g \ .$$

Use this expression for a to find T:

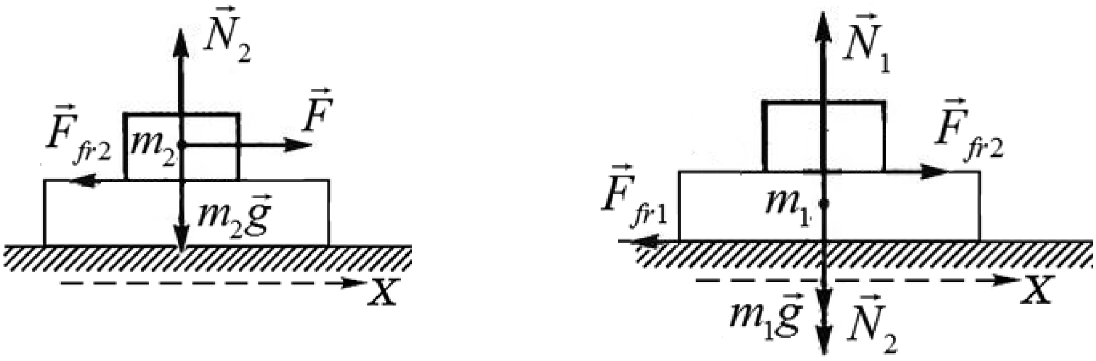
$$T = m_1 \left(\frac{m_2 \sin\alpha - m_1}{m_1 + m_2} \cdot g + g \right) = m_1 m_2 \left(\frac{\sin\alpha + 1}{m_1 + m_2} \right) g \ .$$

The block m_2 accelerates down the incline only if $m_2 \sin\alpha > m_1$. Otherwise, the acceleration is up the incline for the block m_2 and downward for the block m_1.

Example 2.4 Two bodies in contact with each other

A block of mass m_1 = 2 kg is placed on the horizontal surface of the table. Coefficient of the kinetic friction between the block and the surface is μ_1 = 0.2. A second block of mass m_2 = 8 kg is placed upon the first block. Coefficient of the kinetic friction between the blocks is μ_2 = 0.3. A constant horizontal force F is applied to m_2. Find the magnitude of this force when the upper block starts to slide relative to the bottom block.

Solution:



(a) (b)

Figure 2.3.

As the upper block is heavier, the force applied to it causes the motion of the bottom block as well due to the friction between them. The both blocks must experience the same acceleration because they are in contact with each other and remain in contact throughout the motion.

When the blocks start to slide against each other we should consider each of the two blocks individually by categorizing each as an object under a net force. Let's construct a diagram of forces acting on the object for each block as shown in Figures 2.3a and 2.3b.

First, apply Newton's second law to m_2 (Figure 2.3a). The force $\vec{F}$, gravitational force $m_2\vec{g}$, friction force against the surface of the bottom block $\vec{F}_{fr2}$ and normal force against the bottom block $\vec{N}_2$ are acting on it.

$$\vec{F} + m_2\vec{g} + \vec{F}_{fr2} + \vec{N}_2 = m_2\vec{a} .$$

In the component form:

$$\begin{cases} Ox: m_2 a = F - F_{fr2} \\ Oy: 0 = -m_2 g + N_2 \end{cases} .$$

We see, that the normal force acting on m_2 is $N_2 = m_2 g$, and the friction force on the upper block sliding against the bottom block is

$$F_{fr2} = \mu_2 N_2 = \mu_2 m_2 g .$$

Now we can express the force F:

$$F = m_2 a + F_{fr2} = m_2 a + \mu_2 m_2 g \ .$$

To find the unknown acceleration a we now need to consider the motion of the bottom block (Figure 2.3b). Gravitational force $m_1\vec{g}$, friction force against the surface of the table $\vec{F}_{fr1}$ and normal force against the table $\vec{N}_1$ are acting on it. However, as the bottom block is in contact with the upper block, the third Newton's law of action and reaction must be applied to it. The blocks interact through the friction between them, and the bottom block starts to move because the friction force $\vec{F}_{fr2}$ is exerted on it by the upper block. Moreover, the surface of the bottom block causes the normal force on the upper block, and the same force $\vec{N}_2$ of the opposite direction is exerted by the upper block on the bottom block. Considering all these forces, we can apply Newton's second law to the bottom block:

$$m_1\vec{g} + \vec{F}_{fr1} + \vec{F}_{fr2} + \vec{N}_1 + \vec{N}_2 = m_1\vec{a} \ .$$

In the component form:

$$\begin{cases} Ox: m_1 a = -F_{fr1} + F_{fr2} \\ Oy: 0 = -m_1 g + N_1 - N_2 \end{cases} .$$

From the second equation we find that the normal force acting on bottom block against the surface of the table is $N_1 = m_1 g + N_2 = g(m_1 + m_2)$, because the upper block is pressing on the bottom block. Then the friction force on the bottom block sliding on the table is

$$F_{fr1} = \mu_1 N_1 = \mu_1 (m_1 + m_2) g \ .$$

We can substitute this formula into the expression to find a:

$$a = \frac{-F_{fr1} + F_{fr2}}{m_1} = \frac{-\mu_1 (m_1 + m_2) g + \mu_2 m_2 g}{m_1} .$$

Then the force F

$$F = m_2 \frac{-\mu_1(m_1+m_2)g + \mu_2 m_2 g}{m_1} + \mu_2 m_2 g = g\frac{m_2}{m_1}(\mu_2 - \mu_1)(m_1+m_2).$$

Calculations:

$$F = 9.8\frac{8}{2}(0.3-0.2)(2+8) = 39.2\,(\text{N})$$

Problems

1. Besides the gravitational force, a 2.80-kg object is subjected to one other constant force. The object starts from rest and in 1.20 s experiences a displacement of $(4.2\vec{i} - 3.3\vec{j})$ m, where the direction of $\vec{j}$ is the upward vertical direction. Determine the other force.
2. A force $\vec{F}$ applied to an object of mass m_1 produces an acceleration of 3.00 m/s^2. The same force applied to a second object of mass m_2 produces an acceleration of 1.00 m/s^2. (a) What is the value of the ratio m_1/m_2? (b) If m_1 and m_2 are combined into one object, find its acceleration under the action of the force $\vec{F}$.
3. Two forces $\vec{F_1}$ and $\vec{F_2}$ act on a 5.00-kg object. Taking F_1 = 20.0 N and F_2 = 15.0 N, find the accelerations of the object if the vectors of the forces make an angle of (a) 90^0 (b) 60^0 with respect to each other.
4. A 15.0-kg block rests on the floor. (a) What force does the floor exert on the block? (b) A rope is tied to the block and is run vertically over a pulley. The other end is attached to a free-hanging 10.0-kg object. What now is the force exerted by the floor on the 15.0-kg block? (c) If the 10.0-kg object in part (b) is replaced with a 20.0-kg object, what is the force exerted by the floor on the 15.0-kg block?
5. Three forces acting on an object are given by $\vec{F_1} = (-2\vec{i} + 2\vec{j})$ N, $\vec{F_2} = (5\vec{i} - 3\vec{j})$ N, $\vec{F_3} = (-45\vec{i})$ N. The object experiences an acceleration of magnitude 3.75 m/s^2. (a) What is the direction of the acceleration? (b) What is the mass of the object? (c) If the object is initially at rest, what is its speed after 10.0 s? (d) What are the velocity components of the object after 10.0 s?

6. A block slides down a frictionless plane having an inclination of $\alpha = 15.0°$. The block starts from rest at the top, and the length of the incline is 2.00 m. (a) Draw a free-body diagram of the block. Find (b) the acceleration of the block and (c) its speed when it reaches the bottom of the incline.

7. A 3.00-kg object is moving in a plane, with its x and y coordinates given by $x = 5t^2 - 1$ and $y = 3t^3 + 2$, where x and y are in meters and t is in seconds. Find the magnitude of the net force acting on this object at $t = 2.00$ s.

8. The distance between two telephone poles is 50.0 m. When a 1.00-kg bird lands on the telephone wire midway between the poles, the wire sags 0.200 m. (a) Draw a free-body diagram of the bird. (b) How much tension does the bird produce in the wire? Ignore the weight of the wire.

9. A bag of cement of mass 32.5 kg hangs in equilibrium from three wires as suggested in Figure 2.4. Two of the wires make angles $\theta_1 = 60.0°$ and $\theta_2 = 40.0°$ with the horizontal. Assuming the system is in equilibrium, find the tensions T_1, T_2, and T_3 in the wires.

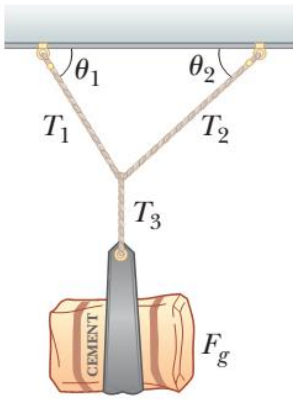


Figure 2.4.

10. An object of mass $m_1 = 5.00$ kg placed on a frictionless, horizontal table is connected to a string that passes over a pulley and then is fastened to a hanging object of mass $m_2 = 9.00$ kg as shown in Figure 2.5. (a) Draw free-body diagrams of both objects. Find (b) the magnitude of the acceleration of the objects and (c) the tension in the string.

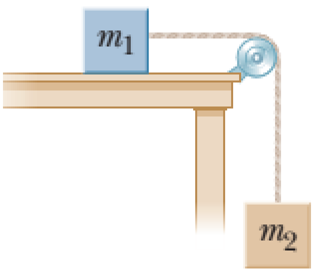


Figure 2.5.

11. Two objects are connected by a light string that passes over a frictionless pulley as shown in Figure 2.6. Assume the incline is frictionless and take m_1 = 2.00 kg, m_2 = 6.00 kg, and θ = 55.0°. (a) Draw free-body diagrams of both objects. Find (b) the magnitude of the acceleration of the objects, (c) the tension in the string, and (d) the speed of each object 2.00 s after it is released from rest.

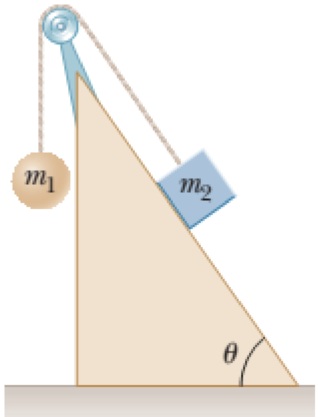


Figure 2.6

12. In the system shown in Figure 2.7, a horizontal force F_x acts on an object of mass m_2 = 8.00 kg. The horizontal surface is frictionless. Consider the acceleration of the sliding object as a function of F_x.

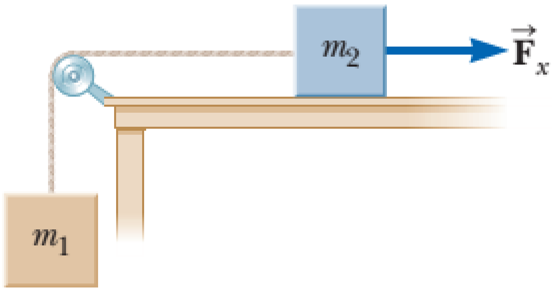


Figure 2.7.

(a) For what values of F_x does the object of mass m_1 = 2.00 kg accelerate upward?
(b) For what values of F_x is the tension in the cord zero?

13. A rifle bullet with a mass of 12.0 g traveling toward the right at 260 m/s strikes a large bag of sand and penetrates it to a depth of 23.0 cm. Determine the magnitude and direction of the friction force (assumed constant) that acts on the bullet.

14. A car is traveling at 50.0 km/h on a horizontal highway. What is the stopping distance when the surface is dry and μ = 0.600?

15. A 25.0-kg block is initially at rest on a horizontal surface. A horizontal force of 75.0 N is required to set the block in motion, after which a horizontal force of 60.0 N is required to keep the block moving with constant speed. Find (a) the coefficient of static friction and (b) the coefficient of kinetic friction between the block and the surface.

16. A 3.00-kg block starts from rest at the top of a 30.0° incline and slides a distance of 2.00 m down the incline in 1.50 s. Find (a) the magnitude of the acceleration of the block, (b) the coefficient of kinetic friction between block and plane, (c) the speed of the block after it has slid 2.00 m.

17. A woman at an airport is towing her 20.0-kg suitcase at constant speed by pulling on a strap at an angle θ above the horizontal. She pulls on the strap with a 35.0-N force, and the friction force on the suitcase is 20.0 N. (a) Draw a free-body diagram of the suitcase. (b) What angle does the strap make with the horizontal? (c) What is the magnitude of the normal force that the ground exerts on the suitcase?

18. A 9.00-kg hanging object is connected by a light, inextensible cord over a light, frictionless pulley to a 5.00-kg block that is sliding on a flat table (Fig. 2.5). Taking the coefficient of kinetic friction as 0.200, find the tension in the string.

19. Three objects are connected on a table as shown in Figure 2.8. The coefficient of kinetic friction between the block of mass m_2 and the table is 0.350.

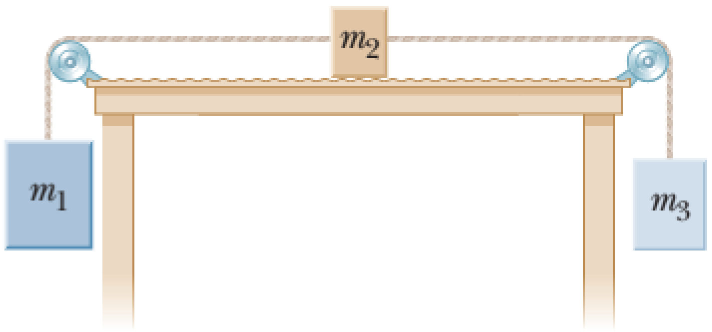


Figure 2.8.

The objects have masses of m_1 = 4.00 kg, m_2 = 1.00 kg, and m_3 = 2.00 kg, and the pulleys are frictionless. (a) Draw a free body diagram of each object. (b) Determine the acceleration of each object, including its direction. (c) Determine the tensions in the two cords. (d) If the tabletop were smooth, would the tensions increase, decrease, or remain the same?

20. Two blocks connected by a rope of negligible mass are being dragged by a horizontal force (Fig. 2.9).

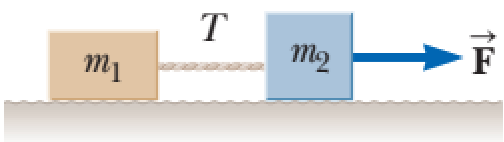


Figure 2.9.

Suppose F = 68.0 N, m_1 = 12.0 kg, m_2 = 18.0 kg, and the coefficient of kinetic friction between each block and the surface is 0.100. (a) Draw a free-body diagram for each block. Determine (b) the acceleration of the system and (c) the tension T in the rope.

21. A block of mass m = 2.00 kg is released from rest at h = 0.500 m above the surface of a table, at the top of a θ = 30.0° incline as shown in Figure 2.10. The frictionless incline is fixed on a table of height H = 2.00 m. (a) Determine the acceleration of the block as it slides down the incline.

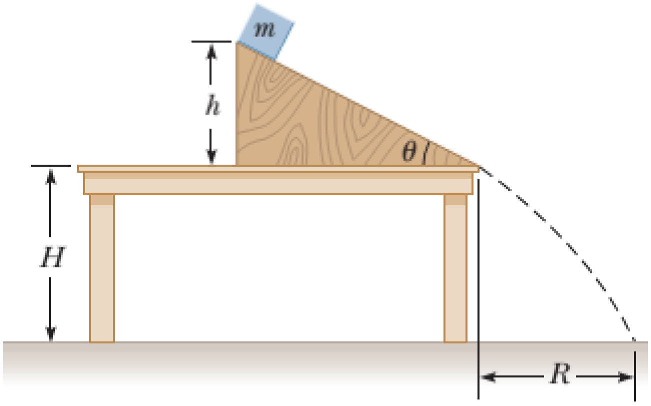


Figure 2.10.

(b) What is the velocity of the block as it leaves the incline? (c) How far from the table will the block hit the floor? (d) What time interval elapses between when the block is released and when it hits the floor? (e) Does the mass of the block affect any of the above calculations?

22. Calculate the force required to pull a copper ball of radius 2.00 cm upward through a fluid at the constant speed 9.00 cm/s. Take the drag force to be proportional to the speed, with proportionality constant 0.950 kg/s. Ignore the buoyant force.

Topic 3. Linear momentum. The law of conservation of linear momentum.

Example 3.1. Impulse

In a particular crash test, a car of mass 1500 kg collides with a wall. The initial and final speeds of the car are v_i = 15 m/s and v_f = 2.6 m/s, respectively. If the collision lasts 0.150 s, find the impulse caused by the collision and the average net force exerted on the car.

Solution:

The collision time is short, so we can imagine the car being brought to rest very rapidly and then moving back in the opposite direction with a reduced speed.

Then the change in the linear momentum of the car is

$$\Delta\vec{p} = \vec{p}_i - \vec{p}_f = m(\vec{v}_i - \vec{v}_f),$$

and its magnitude, considering the opposite directions of the velocity before and after collision,

$$\Delta p = m(v_i - (-v_f)) = m(v_i + v_f).$$

Let us assume the net force exerted on the car by the wall and friction from the ground is large compared with other forces on the car (such as air resistance). Furthermore, the gravitational force and the normal force exerted by the road on the car are perpendicular to the motion and therefore do not affect the horizontal momentum.

Therefore, we categorize the problem as one in which we can apply the impulse approximation in the horizontal direction. We also see that the car's momentum changes due to an impulse from the environment.

Therefore, the impulse applied to the car equals the change in its linear momentum

$$F\Delta t = \Delta p = m(v_i + v_f).$$

The average net force exerted on the car:

$$F = \frac{\Delta p}{\Delta t}.$$

Calculations:

$$F\Delta t = 1500(15 + 2.6) = 26.4 \cdot 10^3 \,(\text{N·s})$$

$$F = \frac{26.4 \cdot 10^3}{0.15} = 17.6 \cdot 10^4 \ (\text{N})$$

Example 3.2. The law of conservation of linear momentum

A ball of mass m = 0.3 kg, moving with a velocity v = 10 m/s, elastically hits a smooth stationary wall so that its velocity is directed at an angle α = 30° to the surface normal. Determine the momentum p obtained by the wall.

Solution:

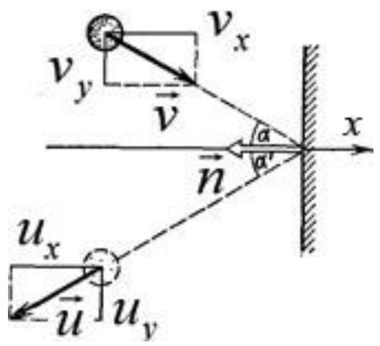


Figure 3.1.

First, we analyze condition of the problem. The wall is stationary, so the reference frame associated with it is inertial. The collision is elastic; therefore, we can use the law of conservation of mechanical energy. As far as the mass of the wall is much larger than the mass of the ball, the absolute values of velocities of the ball $|v|$ before and $|u|$ after the impact must be equal.

To determine the momentum obtained by the wall, we use the law of conservation of the linear momentum:

$$\vec{p}_1 = \vec{p}_1{}' + \vec{p},$$

where $\vec{p}_1 = m\vec{v}$ and $\vec{p}_1{}' = m\vec{u}$ are the momentums of the ball before and after the impact, $\vec{p}$ is the momentum received by the wall. Let's write projection of this equation onto the coordinate axes Ox and Oy (see Figure 3.1):

Ox: $mv\cos\alpha = -mu\cos\alpha' + p$;

Oy: $mv\sin\alpha = mu\sin\alpha'$.

Let us show that the angle of reflection α' is equal to the angle of incidence α. Since the wall is smooth, then projections of the vectors $\vec{v}$ and $\vec{u}$ onto the Oy axis must be equal: $u_y = v_y$. Taking into account that $|v| = |u|$, we obtain $\sin\alpha = \sin\alpha'$, hence $\alpha' = \alpha$. Then, $mv\cos\alpha = -mv\cos\alpha + p$.

Hence the momentum received by the wall, $\vec{p}$, is co-directional with the Ox axis and its magnitude is $p = 2mv\cos\alpha$.

Let's perform the calculations:

$p = 2 \cdot 0.3 \cdot 10 \cdot \frac{\sqrt{3}}{2}$ kg·m/s = 5.20 kg·m/s.

Example 3.3. Relative motion

A boat of length L and mass M is at rest on the calm pond. The boat is perpendicular to the shore, facing it with its bow. A man of mass m is standing at the stern. At what distance s does the boat approach the shore if the man moves from the stern to the bow of the boat? The resistance is negligible.

Solution:

Let's assume that the man walks at a constant speed. In such case the boat moves uniformly too. Therefore, the displacement of the boat relative to the shore is determined by the formula

$s = vt,$

where v is the speed of the boat relative to the shore; t is the time of motion of the man and the boat. The direction of motion of the man is taken as positive.

We find the speed v of the boat using the law of conservation of the linear momentum. Since, according to the problem condition, the man-boat system was initially at rest relative to the shore, then

$Mv + mu = 0,$

where v is the boat's speed relative to the shore, u is the man's speed relative to the shore.

However, the man is moving in the reference frame (boat) which is moving relative to the shore. According to the Galilean velocity addition law,

$$u = v - \frac{L}{t},$$

where u is the man's speed relative to the “shore” frame, v is the speed of the “boat” frame relative to the “shore” frame, $\frac{L}{t}$ is the man's speed relative to the “boat” frame; the negative sign indicates that the velocities of the man and the boat have opposite direction.

Then we find that

$$Mv + m\left(v - \frac{L}{t}\right) = 0; \qquad v = \frac{mL}{(m+M)t}.$$

The time of the boat's motion is equal to the time of the man's motion across the boat, so the displacement of the boat is

$$s = vt = \frac{mL}{(m+M)}.$$

Example 3.4. Collision of two bodies

A 1500-kg car traveling east with a speed of 25.0 m/s collides at an intersection with a 2500-kg truck traveling north at a speed of 20.0 m/s. Find the direction and magnitude of the velocity of the wreckage after the collision, assuming the vehicles stick together after the collision.

Solution:

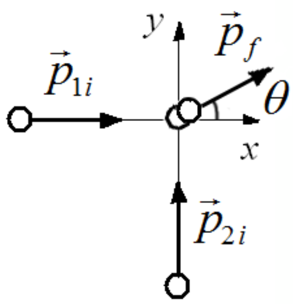


Figure 3.2.

Let's model the vehicles as particles. The collision is perfectly inelastic because the car and the truck stick together after the collision. Because we consider moments immediately before and immediately after the collision as defining our time interval, we ignore the small effect that friction would have on the wheels of the vehicles and model the system of two vehicles as isolated in terms of momentum. Then, according to the law of conversation of the linear momentum of the system, the initial and final momenta of the system must be equal:

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_f,$$

where $\vec{p}_{1i} = m_1\vec{v}_{1i}$ is the initial momentum of the car, $\vec{p}_{2i} = m_2\vec{v}_{2i}$ is the initial momentum of the truck, $\vec{p}_f = (m_1 + m_2)\vec{v}_f$ is the momentum of the wreckage after the collision (see Figure 3.2).

Let us choose east to be along the positive x direction and north to be along the positive y direction and assume that the wreckage moves at an angle θ with respect to the x axis. Then we can write the law of conversation of the linear momentum in projections onto the coordinate axes:

$$\begin{cases} Ox: m_1 v_{1i} = (m_1 + m_2) v_f \cos\theta \\ Oy: m_2 v_{2i} = (m_1 + m_2) v_f \sin\theta \end{cases}.$$

By dividing the *Oy* projection on the *Ox* projection we obtain

$$tg\theta = \frac{m_2 v_{2i}}{m_1 v_{1i}}; \qquad \theta = arctg\left(\frac{m_2 v_{2i}}{m_1 v_{1i}}\right).$$

Knowing the value of the angle θ we can find the final speed as

$$v_f = \frac{m_2 v_{2i}}{(m_1 + m_2)\sin\theta}.$$

Calculations:

$$\theta = arctg\left(\frac{2500\cdot 20}{1500\cdot 25}\right) \approx 53.1^0;\ v_f = \frac{2500\cdot 20}{(1500+2500)\sin 53.1^0} \approx 15.6\,\text{(m/s)}$$

Problems

1. A car of mass *m* traveling at speed *v* crashes into the rear of a truck of mass 2*m* that is at rest and in neutral at an intersection. If the collision is perfectly inelastic, what is the speed of the combined car and truck after the collision?
2. A 2-kg object moving to the right with a speed of 4 m/s makes a head-on, elastic collision with a 1-kg object that is initially at rest. Find the velocity of the 1-kg object after the collision.
3. A 57.0-g tennis ball is traveling straight at a player at 21.0 m/s. The player volleys the ball straight back at 25.0 m/s. If the ball remains in contact with the racket for 0.060 0 s, what average force acts on the ball?
4. At one instant, a 17.5-kg sled is moving over a horizontal surface of snow at 3.50 m/s. After 8.75 s has elapsed, the sled stops. Use a momentum approach to find the average friction force acting on the sled while it was moving.
5. A 45.0-kg girl is standing on a 150-kg plank. Both are originally at rest on a frozen lake that constitutes a frictionless, flat surface. The girl begins to walk in the direction along the plank at a constant speed of 1.50 m/s relative to the plank. (a) What is the velocity of the plank relative to the ice surface? (b) What is the girl's velocity relative to the ice surface?
6. A 1 200-kg car traveling initially at v_{Ci} = 25.0 m/s in an easterly direction crashes into the back of a 9 000-kg truck moving in the same direction at v_{Ti} = 20.0 m/s. The velocity of the car immediately after the collision is v_{Cf} = 18.0 m/s to the east. What is the velocity of the truck immediately after the collision?
7. A 10.0-g bullet is fired into a stationary block of wood having mass m 55.00 kg. The bullet imbeds into the block. The speed of the bullet-plus-wood combination

immediately after the collision is 0.600 m/s. What was the original speed of the bullet?

8. Two automobiles of equal mass approach an intersection. One vehicle is traveling with speed 13.0 m/s toward the east, and the other is traveling north with speed v_{2i}. Neither driver sees the other. The vehicles collide in the intersection and stick together, leaving parallel skid marks at an angle of 55.0^0 north of east. The speed limit for both roads is 60 km/h, and the driver of the northward-moving vehicle claims he was within the speed limit when the collision occurred. Is he telling the truth?

9. An object of mass 3.00 kg, moving with an initial velocity of $5\vec{i}$ m/s, collides with and sticks to an object of mass 2.00 kg with an initial velocity of $-3\vec{j}$ m/s. Find the final velocity of the composite object.

10. A billiard ball moving at 5.00 m/s strikes a stationary ball of the same mass. After the collision, the first ball moves at 4.33 m/s at an angle of 30.0^0 with respect to the original line of motion. Assuming an elastic collision (and ignoring friction and rotational motion), find the struck ball's velocity after the collision.

11. A 3.00-kg steel ball strikes a wall with a speed of 10.0 m/s at an angle of $\theta = 60.0^0$ with the surface. It bounces off with the same speed and angle (Fig. 3.3). If the ball is in contact with the wall for 0.200 s, what is the average force exerted by the wall on the ball?

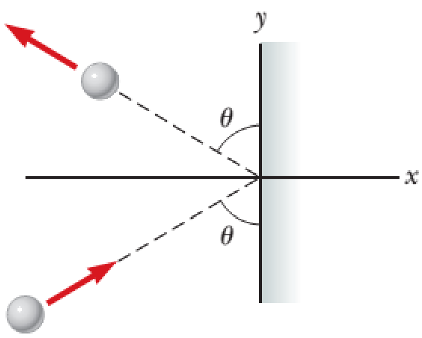


Figure 3.3.

Topic 4. Jet motion. Non-inertial reference frames.

Example 4.1. Rocket propulsion

A 5000 kg rocket moving in space, far from all other objects, has a speed of $3 \cdot 10^3$ m/s relative to the Earth. Its engines are turned on, and fuel is ejected in a direction opposite the rocket's motion at a speed of $5 \cdot 10^3$ m/s relative to the rocket. What is the speed of the rocket relative to the earth once the rocket's mass is reduced to half its mass before ignition? What is the thrust on the rocket and the acceleration due to it if the fuel is burnt at the rate of 50 kg/s?

Solution:

The equation for the speed of the rocket propulsion is

$$v = v_0 + u \ln \frac{M_0}{M},$$

where v_0 is the initial speed of the rocket relative to the earth, u is the speed of the ejected fuel relative to the rocket, $\frac{M_0}{M} = 2$ according to the conditions of the problem.

$$v = 3 \cdot 10^3 + 5 \cdot 10^3 \ln 2 = 6.5 \cdot 10^3 \text{(m/s)}$$

The equation of the rocket propulsion is

$$M\vec{a} = \vec{F} - \vec{u}\frac{dM}{dt},$$

where $\vec{F}$ is the net external force on the system; when the rocket moves in space $\vec{F}$ =0; $\frac{dM}{dt} = 50$ kg/s is the rate at which the fuel is burnt. So,

$$Ma = u\frac{dM}{dt}$$

The thrust on the rocket is

$$Thrust = \left|\vec{u}\frac{dM}{dt}\right|; \qquad Thrust = 5\cdot 10^3 \cdot 50 = 2.5\cdot 10^5 \text{ (N)}$$

Acceleration of the rocket

$$a = \frac{u}{M}\frac{dM}{dt}; \qquad a = \frac{5\cdot 10^3 \cdot 50}{5000} = 50 \text{(m/s}^2\text{)}$$

Example 4.2.

A 3500 kg helicopter with a rotor of diameter 18 m is hanging in the air. What is the speed *u* of the air flow blown down by the rotor? Consider diameter of the flow equal to the diameter of the rotor.

Solution:

The helicopter is hanging in the air due to the thrust exerted by the repelled air flow. The equation of the jet motion is

$$m_h\vec{a} = \vec{F} - \vec{u}\frac{dm_{air}}{dt},$$

where $\vec{F} = m_h\vec{g}$ because the helicopter is located in the gravitational field of the Earth; m_h is the mass of the helicopter; m_{air} is the mass of the air and $\frac{dm_{air}}{dt}$ is change in mass of the air adjacent to the rotor.

As far as the helicopter is hanging at one place and not moving, its acceleration $a = 0$. Then,

$$0 = -m_h g + u\frac{dm_{air}}{dt}.$$

Mass of the air

$$m_{air} = \rho_{air}V = \rho_{air}Sh = \rho_{air}\frac{\pi d^2}{4}h,$$

where ρ_{air} is the density of the air, $V = Sh$ is the volume of the air layer adjacent to the rotor, $S = \frac{\pi d^2}{4}$ is its cross-sectional area. Then,

$$\frac{dm_{air}}{dt} = \frac{d}{dt}\left(\rho_{air}\frac{\pi d^2}{4}h\right) = \rho_{air}\frac{\pi d^2}{4}\frac{dh}{dt} = \rho_{air}\frac{\pi d^2}{4}u,$$

because change in the height of the air layer with time equals to the speed of its flow $\frac{dh}{dt} = u$.

After substitution we obtain

$$m_h g = u\cdot\rho_{air}\frac{\pi d^2}{4}u = \rho_{air}\frac{\pi d^2 u^2}{4},$$

and solving for u:

$$u = \sqrt{\frac{4m_h g}{\rho_{air}\pi d^2}}.$$

Calculations:

$$u = \sqrt{\frac{4\cdot 3500\cdot 9.8}{1.29\pi\cdot 18^2}} \approx 10.2\,\text{(m/s)}$$

Example 4.3.

A trolley with sand is moving along the horizontal surface under a constant horizontal force $\vec{F}$. The sand is pouring down through a hole in the bottom of the trolley at constant speed μ kg/s. Find the speed and acceleration of the trolley at the moment t if at the moment $t = 0$ the trolley is at rest and has a mass m_0. Friction is negligible.

Solution:

As far as the sand is pouring away from the trolley, its mass is changing during motion. The equation of its motion is:

$$m\,\vec{a} = \vec{F}_{net} - \vec{u}\,\frac{dm}{dt},$$

where $\vec{F}_{net} = \vec{F} + m\vec{g}$; $\frac{dm}{dt} = -\mu$ is the rate of changing mass (the negative sign is because the mass is decreasing), and

$$m\,\vec{a} = \vec{F} + m\,\vec{g} + \vec{u}\mu.$$

Let's write this equation in projection onto the direction of motion. As far as the sand is pouring vertically down, projection of its velocity onto the horizontal direction is zero. We obtain

$$m\,a = F; \qquad a = \frac{F}{m}.$$

Let's find mass of the trolley at the moment *t*:

$$\frac{dm}{dt} = -\mu; \qquad dm = -\mu dt.$$

We can take integral of this equality:

$$\int_{m_0}^{m} dm = -\int_0^t \mu dt; \qquad m = m_0 - \mu t.$$

Then, the acceleration of the trolley is

$$a = \frac{F}{m_0 - \mu t}.$$

Speed of the trolley can be found as $v = \int_0^t a dt$ considering that the initial speed of the trolley is zero. Then,

$$v=\int_0^t \frac{F}{m_0-\mu t}dt=-\frac{F}{\mu}\ln(m_0-\mu t)\Big|_0^t=-\frac{F}{\mu}\ln\left(\frac{m_0-\mu t}{m_0}\right).$$

That is,

$$v=\frac{F}{\mu}\ln\left(\frac{m_0}{m_0-\mu t}\right).$$

Example 4.4. Noninertial reference frames. Translational inertial force.

A tanker with kerosene moves with acceleration a = 0.7 m/s^2. What angle α does the kerosene level make with respect to the horizontal?

Solution:

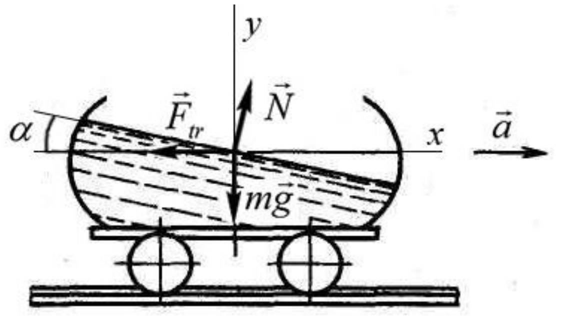


Figure 4.1.

Consider a mass element on the kerosene surface. Normal force exerted by the underlying kerosene and gravitational force are acting on it (see Figure 4.1). However, relative to the reference frame associated with the tanker, the level of the kerosene is at rest and makes angle α with the horizontal. So its acceleration is zero. Therefore, in that noninertial frame we introduce a fictitious force in the horizontal direction to balance the horizontal component of the normal force.

Newton's second law for the element on the kerosene surface can be written as

$$0=\vec{N}+m\vec{g}+\vec{F}_{tr},$$

where $\vec{F}_{tr}=-m\vec{a}$ is the translational inertial force due to the accelerated translational motion of the frame associated with the tanker.

Let's apply Newton's second law in component form:

$$\begin{cases} Ox: 0=N\sin\alpha-ma \\ Oy: 0=N\cos\alpha-mg \end{cases}; \qquad \begin{cases} ma=N\sin\alpha \\ mg=N\cos\alpha \end{cases}.$$

By finding the ratio of these expressions, we obtain

$$tg\alpha = \frac{a}{g}; \qquad \alpha = arctg\left(\frac{a}{g}\right)$$

Calculations:

$$\alpha = arctg\left(\frac{0.7}{9.8}\right) \approx 4^0.$$

Example 4.5. Noninertial reference frames. Centrifugal inertial force.

A vessel with water rotates at a frequency of 2 s^{-1} about a vertical axis. Water surface takes form of paraboloid. What angle α does the water surface make with the horizontal at distance R = 5 cm from the axis of rotation?

Solution:

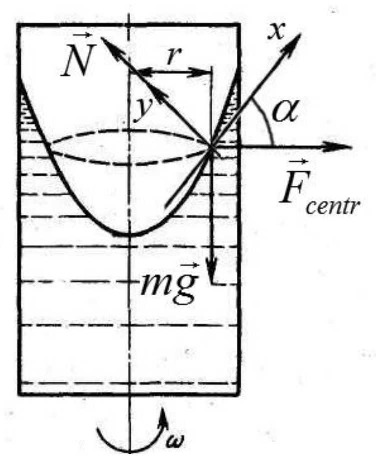


Figure 4.2.

Consider a mass element on the water surface in the noninertial reference frame associated with the rotating vessel. The mass element is under the action of normal force exerted by the underlying water, gravitational force and centrifugal force resulting from the rotation of the reference frame. Since the mass element is motionless relative to the vessel, the net force must be zero.

Newton's second law for the element on the water surface can be written as

$$0 = \vec{N} + m\vec{g} + \vec{F}_{centr},$$

where $\vec{F}_{centr} = m\omega^2 R\vec{n}$ is the centrifugal inertial force due to the rotation of the frame associated with the vessel; $\omega = 2\pi\upsilon$ is the angular speed of rotation, R is the distance from the axis of rotation.

Let's apply Newton's second law in component form choosing the *Ox* axis directed tangent to the surface (see Figure 4.2):

$$Ox: 0 = 0 - mg\sin\alpha + F_{centr}\cos\alpha;$$

$$mg\sin\alpha = m(2\pi\upsilon)^2 R\cos\alpha;$$

$$tg\alpha = \frac{4\pi^2\upsilon^2 R}{g}; \qquad \alpha = arctg\left(\frac{4\pi^2\upsilon^2 R}{g}\right).$$

Calculations:

$$\alpha = arctg\left(\frac{4\pi^2 \cdot 2^2 \cdot 0.05}{9.8}\right) \approx 40^0.$$

Example 4.6. Noninertial reference frames. Coriolis inertial force.

A small clutch of mass *m* freely slides along a smooth horizontal rod which is rotated at a constant angular velocity $\vec{\omega}$ about a fixed vertical axis passing through one of its ends. Find the horizontal component of the force exerted on the clutch by the rod at the moment when it is at a distance *r* from the axis of rotation. (At the initial moment the clutch is directly near the axis and has a negligible speed.)

Solution:

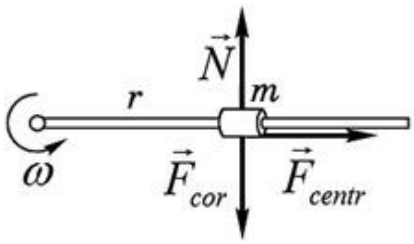


Figure 4.3.

Consider motion of the clutch in the rotating reference frame associated with the rod. In that frame the clutch moves along a straight line, which means that the force $\vec{N}$ exerted on the clutch by the rod is balanced by the Coriolis force (see Figure 4.3, top view):

$$\vec{N} = -\vec{F}_{cor} = -2m[\vec{v}, \vec{\omega}].$$

We need to find the speed v of the clutch relative to the rod. In our frame of reference, in the direction along the rod the clutch moves under the action of the centrifugal inertial force. So the Newton's second law may be written as:

$$ma = F_{centr};$$

$$m\frac{dv}{dt} = m\omega^2 r; \qquad \frac{dv}{dt} = \omega^2 r$$

Let's express this equation in the variables v and r:

$$\frac{dv}{dt}\cdot\frac{dr}{dr} = \frac{dv}{dr}\cdot\frac{dr}{dt} = \frac{dv}{dr}\cdot v.$$

Then

$$\frac{dv}{dr}\cdot v = \omega^2 r; \qquad vdv = \omega^2 rdr.$$

Integrating the last equation with the initial conditions ($v_0 = 0$, $r_0 = 0$), we find

$$\frac{v^2}{2} = \frac{\omega^2 r^2}{2};$$

$$v = \omega r.$$

In vector form, $\vec{v} = \omega\vec{r}$. So, the force

$$\vec{N} = 2m\left[\vec{\omega}, \omega\vec{r}\right] = 2m\omega\left[\vec{\omega}, \vec{r}\right];$$

$$N = 2m\omega^2 r.$$

Problems

Jet motion

1. A model rocket engine has an average thrust of 5.26 N. It has an initial mass of 25.5 g, which includes fuel mass of 12.7 g. The duration of its burn is 1.90 s. (a) What is the average exhaust speed of the engine? (b) This engine is placed in a rocket body of mass 53.5 g. What is the final velocity of the rocket if it were to be fired from rest

in outer space by an astronaut on a spacewalk? Assume the fuel burns at a constant rate.

2. The first stage of a Saturn V space vehicle consumed fuel and oxidizer at the rate of 15000 kg/s with an exhaust speed of 2600 m/s. (a) Calculate the thrust produced by this engine. (b) Find the acceleration the vehicle had just as it lifted off the launch pad on the Earth, taking the vehicle's initial mass as $3.00 \cdot 10^6$kg.
3. A rocket for use in deep space is to be capable of boosting a total load (payload plus rocket frame and engine) of 3.00 tons to a speed of 10 000 m/s. It has an engine and fuel designed to produce an exhaust speed of 2000 m/s. How much fuel plus oxidizer is required?

Accelerated frames

1. Two blocks, each of mass m = 3.50 kg, are hung from the ceiling of an elevator as in Figure 4.4. (a) If the elevator moves with an upward acceleration of magnitude a = 1.60 m/s^2, find the tensions T_1 and T_2 in the upper and lower strings. (b) If the strings can withstand a maximum tension of 85.0 N, what maximum acceleration can the elevator have before a string breaks?

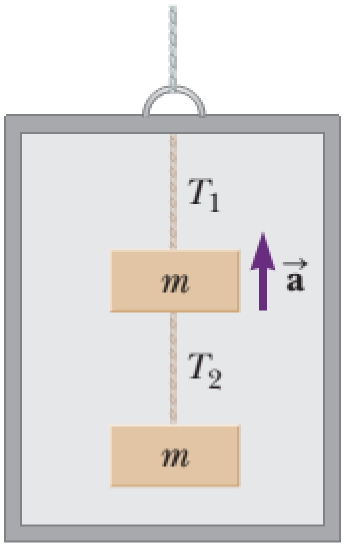


Figure 4.4

2. A light string can support a stationary hanging load of 25.0 kg before breaking. An object of mass m = 3.00 kg attached to the string rotates on a frictionless, horizontal table in a circle of radius r = 0.800 m, and the other end of the string is held fixed as in Figure 4.5. What range of speeds can the object have before the string breaks?

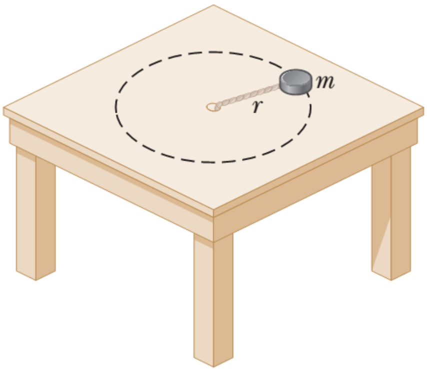


Figure 4.5.

3. Consider a conical pendulum (Fig. 4.6) with a bob of mass m = 80.0 kg on a string of length L = 10.0 m that makes an angle of $\theta = 5.00^0$ with the vertical. Determine (a) the period of rotation and (b) the radial acceleration of the bob.

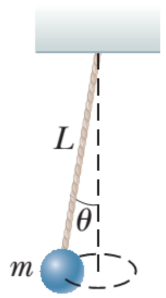


Figure 4.6.

4. A 40.0-kg child swings in a swing supported by two chains, each 3.00 m long. The tension in each chain at the lowest point is 350 N. Find (a) the child's speed at the lowest point and (b) the force exerted by the seat on the child at the lowest point. (Ignore the mass of the seat.)

5. An adventurous archeologist (m 585.0 kg) tries to cross a river by swinging from a vine. The vine is 10.0 m long, and his speed at the bottom of the swing is 8.00 m/s. The archeologist doesn't know that the vine has a breaking strength of 1 000 N. Does he make it across the river without falling in?

6. An object of mass m = 0.500 kg is suspended from the ceiling of an accelerating truck as shown in Figure 4.7. Taking a = 3.00 m/s^2, find (a) the angle θ that the string makes with the vertical and (b) the tension T in the string.

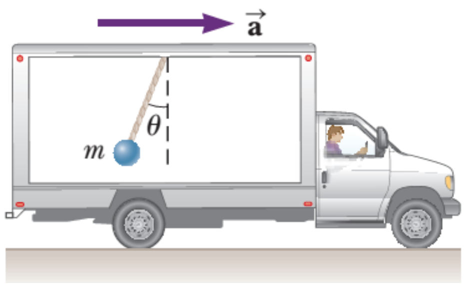


Figure 4.7.

7. A truck is moving with constant acceleration a up a hill that makes an angle ϕ with the horizontal as in Figure 4.8. A small sphere of mass m is suspended from the ceiling of the truck by a light cord. If the pendulum makes a constant angle θ with the perpendicular to the ceiling, what is a?

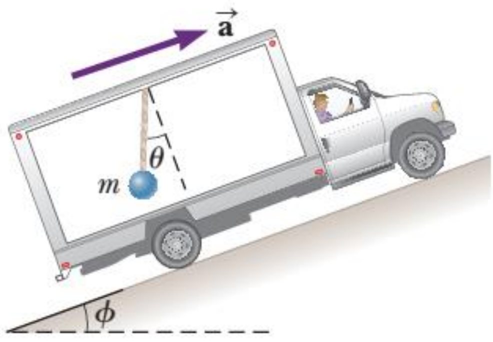


Figure 4.8.

8. A small container of water is placed on a turntable inside a microwave oven, at a radius of 12.0 cm from the center. The turntable rotates steadily, turning one revolution in each 7.25 s. What angle does the water surface make with the horizontal?

9. A child's toy consists of a small wedge that has an acute angle θ (Fig. 4.9). The sloping side of the wedge is frictionless, and an object of mass m on it remains at constant height if the wedge is spun at a certain constant speed. The wedge is spun by rotating, as an axis, a vertical rod that is firmly attached to the wedge at the bottom end. Find the speed of the object when it sits at rest at a point at distance L up along the wedge.

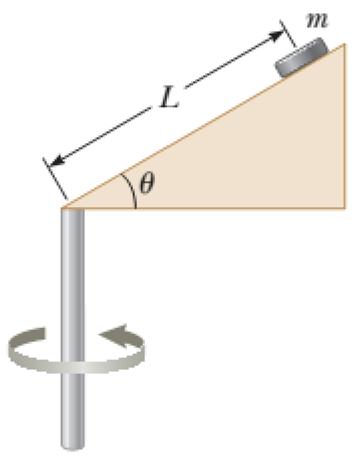


Figure 4.9.

Topic 5. Energy, work, power. The law of conservation of the total mechanical energy.

Example 5.1. Collision of two bodies.

Two balls with masses m_1 = 2.5 kg and m_2 = 1.5 kg move towards each other at speeds v_1 = 6 m/s and v_2 = 2 m/s. Determine: 1) the speed u of the balls after the impact; 2) the kinetic energies of the balls T_1 before and T_2 after the impact; 3) the fraction of the kinetic energy of the balls that is converted into the internal energy. The impact is considered as direct and inelastic.

Solution

1) As the impact is inelastic, the balls after the impact move together at the same velocity $\vec{u}$. Let us determine this velocity according to the law of conservation of the linear momentum:

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{u}.$$

Let's take the direction of the velocity of the first ball as positive. Then, since the balls move along a straight line, this law can be written in a scalar form:

$$m_1v_1 - m_2v_2 = (m_1 + m_2)u,$$

and,

$$u = \frac{m_1v_1 - m_2v_2}{m_1 + m_2}.$$

The calculations:

u = (2.5·6 – 1.5·2)/(2.5 + 1.5) m/s = 3 m/s.

2) The kinetic energies of the balls before and after the impact are determined by the formulas

$$T_1 = \frac{m_1v_1^2}{2} + \frac{m_2v_2^2}{2}; \qquad T_2 = \frac{(m_1 + m_2)u^2}{2}.$$

After calculations we obtain

$T_1 = (2.5 \cdot 6^2/2 + 1.5 \cdot 2^2/2)$ J = 48 J;

$T_2 = (2.5 + 1.5)\ 3^2$ J = 18 J.

3) Comparing kinetic energies of the balls before and after the inelastic collision, we see decrease in their kinetic energy, due to which their internal energy has increased. The fraction of the kinetic energy of the balls causing increase in their internal energy can be determined from the relation:

$$w = \frac{T_1 - T_2}{T};\ w = 0.62.$$

Example 5.2. Situations involving kinetic friction.

A 3.00-kg crate slides down a ramp. The ramp is 1.00 m in length and inclined at an angle of 30^0 as shown in Figure 5.1. The crate starts from rest at the top and continues to move a short distance on the horizontal floor after it leaves the ramp. The coefficient of kinetic friction of the ramp and the floor is 0.3. Use energy methods to determine the speed of the crate at the bottom of the ramp. How far does the crate slide on the horizontal floor?

Solution:

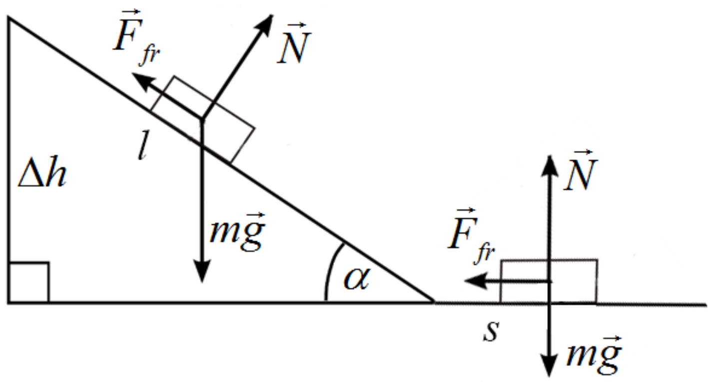


Figure 5.1.

We identify the crate, the surface, and the Earth as the system. The system is categorized as isolated with a nonconservative force of friction acting on the body.

Because the initial speed of the crate is zero, the initial kinetic energy of the system when the crate is at the top of the ramp is zero. If the *y* coordinate is measured from the bottom of the ramp (the final position of the crate, for which we choose the gravitational potential energy of the system to be zero) with the upward direction being positive, then the initial elevation of the crate is $\Delta h = l\sin\alpha$. The potential energy of the system decreases, whereas the kinetic energy increases.

The expression for the total mechanical energy of the system when the crate is at the top:

$$E_{total1} = U = mgl\sin\alpha .$$

When the crate slides down its potential energy transforms into kinetic energy and work against friction. At the bottom of the ramp

$$E_{total2} = K - A_{fr} = \frac{mv^2}{2} - A_{fr},$$

where negative sign of the work indicates that it is done by the system. The work by friction along the ramp is

$$A_{fr} = \int_0^l \vec{F}_{fr} \cdot d\vec{r} = -F_{fr} l$$

(the negative sign appears because the force of friction $\vec{F}_{fr}$ and the displacement vector $d\vec{r}$ of the body are always in opposite directions). So,

$$E_{total2} = \frac{mv^2}{2} + F_{fr} l .$$

By applying the Newton's second law for the body sliding down the incline, we can find the force of friction on the crate

$$F_{fr} = \mu N = \mu mg\cos\alpha .$$

Now we can equate the expressions for the total mechanical energy:

$$E_{total1} = E_{total2};$$

$$mgl\sin\alpha = \frac{mv^2}{2} + \mu mg\cos\alpha \cdot l; \quad \frac{v^2}{2} = gl(\sin\alpha - \mu\cos\alpha);$$

$$v = \sqrt{2gl(\sin\alpha - \mu\cos\alpha)}.$$

Substitute numerical values:

$$v = \sqrt{2 \cdot 9.8 \cdot 1 \cdot (\sin 30^0 - 0.3\cos 30^0)} \approx 2.17 \text{ (m/s)}.$$

Now consider the crate sliding on the horizontal floor. In this case the potential energy of the system remains fixed and the mechanical energy of the system consists only of kinetic energy. We can apply the work- kinetic energy theorem:

$$A_{fr} = K_f - K_i,$$

where the work by friction along the path s before the crate stops is $A_{fr} = -F_{fr}s$; the final kinetic energy of the crate $K_f = 0$; the initial kinetic energy of the crate is its kinetic energy at the bottom of the ramp $K_i = \frac{mv^2}{2} = mgl(\sin\alpha - \mu\cos\alpha)$.

After substitution we obtain

$$-F_{fr}s = 0 - mgl(\sin\alpha - \mu\cos\alpha); \qquad s = \frac{mgl(\sin\alpha - \mu\cos\alpha)}{F_{fr}}.$$

Applying the Newton's second law for the body on the horizontal surface, we can find the force of friction on the crate sliding on the floor

$$F_{fr} = \mu N = \mu mg.$$

Then,

$$s = \frac{mgl(\sin\alpha - \mu\cos\alpha)}{\mu mg} = \frac{l(\sin\alpha - \mu\cos\alpha)}{\mu}.$$

Calculations:

$$s = \frac{1 \cdot (\sin 30^0 - 0.3\cos 30^0)}{0.3} \approx 0.8 \text{ (m)}$$

Example 5.3. Ballistic pendulum

The ballistic pendulum (Figure 5.2) is an apparatus used to measure the speed of a fast-moving projectile such as a bullet. A projectile of mass m_1 is fired into a large block of wood of mass m_2 suspended from some light wires. The projectile embeds in the block, and the entire system swings through a height h. How can we determine the speed of the projectile from a measurement of h?

Solution:

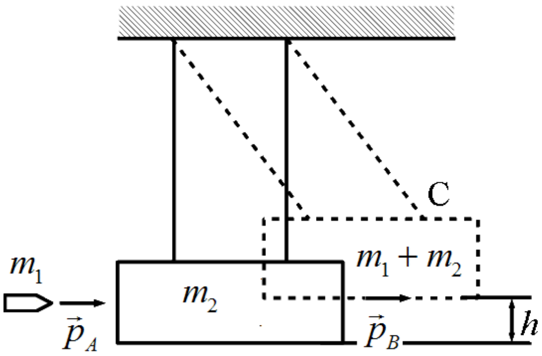


Figure 5.2.

The projectile and the block form an isolated system in terms of momentum if we identify configuration A as immediately before the collision and configuration B as immediately after the collision.

Because the projectile imbeds in the block, we can categorize the collision between them as perfectly inelastic.

For the moments immediately before and after the collision we can use the law of conservation of the linear momentum:

$$\vec{p}_A = \vec{p}_B,$$

where $\vec{p}_A = m_1\vec{v}$ is the momentum of the projectile before the impact; $\vec{p}_B = (m_1 + m_2)\vec{u}$ is the momentum of the block with the embedded projectile after the impact. In trojection onto the direction of motion

$$m_1 v = (m_1 + m_2)u .$$

Then the speed of the projectile–block system immediately after the collision

$$u = \frac{m_1 v}{m_1 + m_2}.$$

For the process during which the projectile–block combination swings upward to height h (ending at a configuration we'll call C), we focus on a different system, that of the projectile, the block, and the Earth. We categorize this part of the problem as one involving an isolated system for energy with no nonconservative forces acting.

We can use the law of conservation of the total mechanical energy. The kinetic energy of the projectile–block combination immediately after the collision is

$$K_B = \frac{(m_1 + m_2)u^2}{2} = \frac{m_1^2 v^2}{2(m_1 + m_2)}.$$

This kinetic energy of the system immediately after the collision is less than the initial kinetic energy of the projectile as is expected in an inelastic collision.

Let's define the gravitational potential energy of the system for configuration B to be zero. Therefore, $U_B = 0$, whereas

$$U_C = (m_1 + m_2)gh.$$

Applying the conservation of mechanical energy principle to the system:

$K_B = U_C$;

$$\frac{m_1^2 v^2}{2(m_1 + m_2)} = (m_1 + m_2)gh,$$

and solving to define the speed of the projectile v:

$$v = \frac{(m_1 + m_2)}{m_1}\sqrt{2gh}.$$

Example 5.4.

A small body A slides off the top of a smooth sphere of radius R. Find the angle θ between the vertical and the position vector of the body drawn from the center of the sphere at the moment when the body comes off the surface of the sphere. Find the speed of the body at that moment (its initial speed is negligibly small).

Solution:

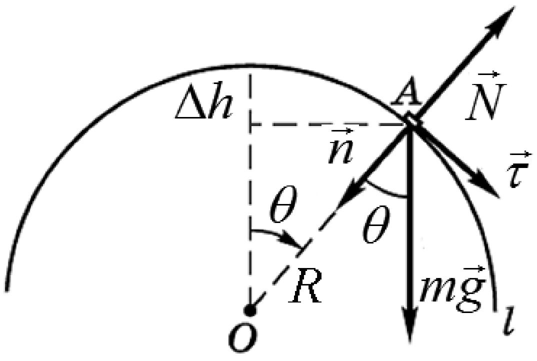


Figure 5.3.

The Newton's second law for the body moving on the smooth surface is:

$$m\vec{a} = m\vec{g} + \vec{N} \ .$$

At the moment when the body comes off the surface, the normal force disappears $\vec{N} = 0$, and then $\vec{a} = \vec{g}$.

Let's take the *Ox* axis tangent to the surface of the sphere at that point, and the *Oy* axis in the radial direction. Then (see Figure 5.3)

$$a_\tau = g\sin\theta; \qquad a_r = g\cos\theta.$$

Radial acceleration $a_r = \dfrac{v^2}{R}$, consequently, the speed of the body at the moment it comes off the surface is

$$v = \sqrt{gR\cos\theta} \ .$$

Let's us the law of conservation of the total mechanical energy for the body. As far as its initial speed is considered zero, at the highest point of the sphere it has only the gravitational potential energy

$$U_{gr} = mg\Delta h ,$$

where the elevation $\Delta h = R - R\cos\theta$ (see Figure 5.3). The potential energy has transformed into the kinetic energy of the body:

$$mg\Delta h = \frac{mv^2}{2} .$$

Then,

$$gR(1-\cos\theta) = \frac{v^2}{2} = \frac{gR\cos\theta}{2} .$$

After transformations we obtain

$$\cos\theta = \frac{2}{3}.$$

So, the speed of the body

$$v = \sqrt{\frac{2}{3} gR}.$$

Problems:

1. A 10.0-g bullet is fired into a 200-g block of wood at rest on a horizontal surface. After impact, the block slides 8.00 m before coming to rest. If the coefficient of friction between the block and the surface is 0.400, what is the speed of the bullet before impact?
2. A wad of sticky clay of mass m is hurled horizontally at a wooden block of mass M initially at rest on a horizontal surface. The clay sticks to the block. After impact, the block slides a distance d before coming to rest. If the coefficient of friction between the block and the surface is μ, what was the speed of the clay immediately before impact?
3. A block of mass m = 2.50 kg is pushed a distance d = 2.20 m along a frictionless, horizontal table by a constant applied force of magnitude F = 16.0 N directed at an angle $\theta = 25.0^0$ below the horizontal. Determine the work done on the block by (a) the applied force, (b) the normal force exerted by the table, (c) the gravitational force, and (d) the net force on the block.
4. Figure 5.4 shows three points in the operation of the ballistic pendulum. The projectile approaches the pendulum in Figure 5.4a.

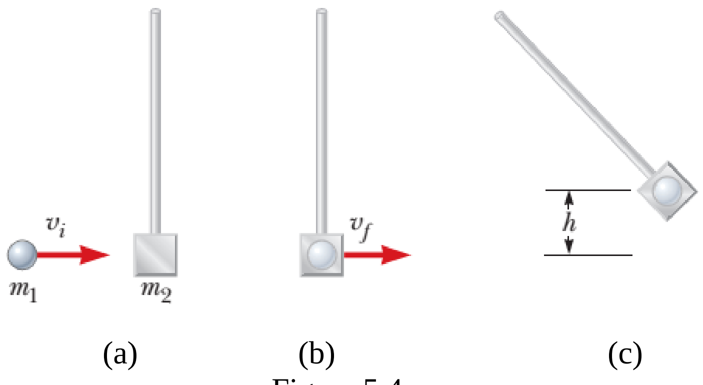


Figure 5.4.

Figure 5.4b shows the situation just after the projectile is captured in the pendulum. In Figure 5.4c, the pendulum arm has swung upward and come to rest at a height h above its initial position. Find the ratio of the kinetic energy of the projectile–pendulum system immediately after the collision to the kinetic energy immediately before.

5. A wooden block of mass M rests on a table over a large hole as in Figure 5.5. A bullet of mass m with an initial velocity of v_i is fired upward into the bottom of the block and remains in the block after the collision. The block and bullet rise to a maximum height of h. Find an expression for the initial velocity of the bullet.

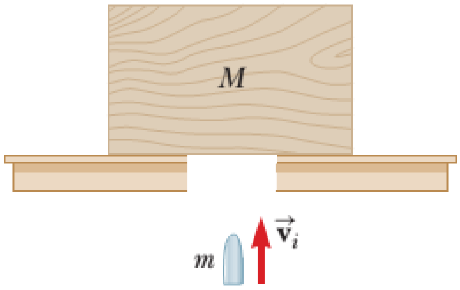


Figure 5.5.

6. Two blocks of masses m_1 = 2.00 kg and m_2 = 4.00 kg are released from rest at a height of h = 5.00 m on a frictionless track as shown in Figure 5.6. When they meet on the level portion of the track, they undergo a head-on, elastic collision. Determine the maximum heights to which m_1 and m_2 rise on the curved portion of the track after the collision.

Figure 5.6.

7. A bullet of mass m is fired into a block of mass M initially at rest at the edge of a frictionless table of height h (Fig. 5.7). The bullet remains in the block, and after impact the block lands a distance d from the bottom of the table.

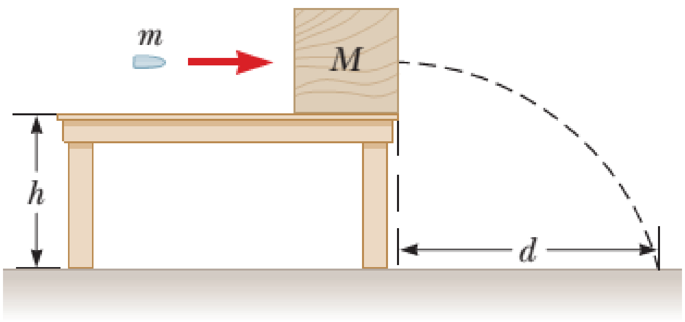


Figure 5.7.

Determine the initial speed of the bullet.

8. George of the Jungle, with mass m, swings on a light vine hanging from a stationary tree branch. A second vine of equal length hangs from the same point, and a gorilla of larger mass M swings in the opposite direction on it. Both vines are horizontal when the primates start from rest at the same moment. George and the gorilla meet at the lowest point of their swings. Each is afraid that one vine will break, so they grab each other and hang on. They swing upward together, reaching a point where the vines make an angle of 35.0^0 with the vertical. Find the value of the ratio m/M.

9. A force $\vec{F} = (6\vec{i} - 2\vec{j})$ N acts on a particle that undergoes a displacement $\Delta\vec{r} = (3\vec{i} + \vec{j})$ m. Find (a) the work done by the force on the particle and (b) the angle between $\vec{F}$ and $\Delta\vec{r}$.

10. A force $\vec{F} = (4x\vec{i} + 3y\vec{j})$, where F is in newtons and x and y are in meters, acts on an object as the object moves in the x direction from the origin to x = 5.00 m. Find the work done by the force on the object.

11. A 100-g bullet is fired from a rifle having a barrel 0.600 m long. Choose the origin to be at the location where the bullet begins to move. Then the force (in newtons) exerted by the expanding gas on the bullet is $15000 + 10000x - 25000x^2$, where x is in meters. Determine the work done by the gas on the bullet as the bullet travels the length of the barrel.

12. A 0.600-kg particle has a speed of 2.00 m/s at point A and kinetic energy of 7.50 J at point B. What is (a) its kinetic energy at A, (b) its speed at B, and (c) the net work done on the particle by external forces as it moves from A to B?

13. A worker pushing a 35.0-kg wooden crate at a constant speed for 12.0 m along a wood floor does 350 J of work by applying a constant horizontal force of magnitude F on the crate. Determine the value of F.

14. A 3.00-kg object has a velocity $(6\vec{i} - \vec{j})$ m/s. (a) What is its kinetic energy at this moment? (b) What is the net work done on the object if its velocity changes to $(8\vec{i} + 4\vec{j})$ m/s.

15. A 7.80-g bullet moving at 575 m/s strikes the hand of a superhero, causing the hand to move 5.50 cm in the direction of the bullet's velocity before stopping. (a) Use work and energy considerations to find the average force that stops the bullet. (b) Assuming the force is constant, determine how much time elapses between the moment the bullet strikes the hand and the moment it stops moving.

16. A baseball outfielder throws a 0.150-kg baseball at a speed of 40.0 m/s and an initial angle of 30.0^0 to the horizontal. What is the kinetic energy of the baseball at the highest point of its trajectory?

17. A 40-kg child is in a swing that is attached to a pair of ropes 2.00 m long. Find the gravitational potential energy of the child–Earth system relative to the child's lowest position when (a) the ropes are horizontal, (b) the ropes make a 30.0^0 angle with the vertical, and (c) the child is at the bottom of the circular arc.

18. The potential energy of a system of two particles separated by a distance r is given by $U(r) = A/r$, where A is a constant. Find the radial force F_r that each particle exerts on the other.

19. A potential energy function for a system in which a two dimensional force acts is of the form $U = 3x^3y - 7x$. Find the force that acts at the point (x, y).

20. An inclined plane of angle $\theta = 20.0^0$ has a spring of force constant $k = 500$ N/m fastened securely at the bottom so that the spring is parallel to the surface as shown in Figure 5.8. A block of mass $m = 2.50$ kg is placed on the plane at a distance $d = 0.300$ m from the spring.

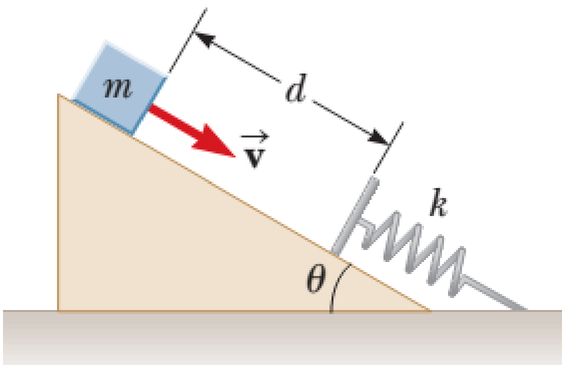


Figure 5.8.

From this position, the block is projected downward toward the spring with speed $v = 0.750$ m/s. By what distance is the spring compressed when the block momentarily comes to rest?

21. A block of mass $m = 2.00$ kg is attached to a spring of force constant $k = 500$ N/m as shown in Figure 5.9. The block is pulled to a position $x_i = 5.00$ cm to the right of equilibrium and released from rest. Find the speed the block has as it passes through equilibrium if

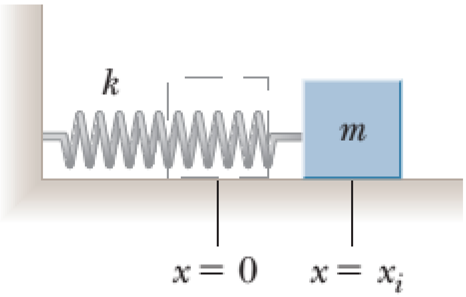


Figure 5.9.

(a) the horizontal surface is frictionless and (b) the coefficient of friction between block and surface is $\mu = 0.350$.

22. A crate of mass 10.0 kg is pulled up a rough incline with an initial speed of 1.50 m/s. The pulling force is 100 N parallel to the incline, which makes an angle of 20.0^0 with the horizontal. The coefficient of kinetic friction is 0.400, and the crate is pulled 5.00 m. (a) How much work is done by the gravitational force on the crate? (b) How much work is done by the 100-N force on the crate? (c) What is the change in kinetic energy of the crate? (d) What is the speed of the crate after being pulled 5.00 m?

23. The coefficient of friction between the block of mass $m_1 = 3.00$ kg and the surface in Figure 5.10 is $\mu = 0.400$. The system starts from rest. What is the speed of the ball of mass $m_2 = 5.00$ kg when it has fallen a distance $h = 1.50$ m?

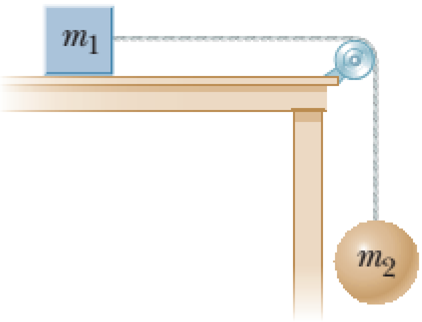


Figure 5.10.

24. A toy cannon uses a spring to project a 5.30-g soft rubber ball. The spring is originally compressed by 5.00 cm and has a force constant of 8.00 N/m. When the cannon is fired, the ball moves 15.0 cm through the horizontal barrel of the cannon, and the barrel exerts a constant friction force of 0.032 N on the ball. (a) With what

speed does the projectile leave the barrel of the cannon? (b) At what point does the ball have maximum speed? (c) What is this maximum speed?

25. An electric scooter has a battery capable of supplying 120 Wh of energy. If friction forces and other losses account for 60.0% of the energy usage, what altitude change can a rider achieve when driving in hilly terrain if the rider and scooter have a combined mass of 89 kg?

26. An older-model car accelerates from 0 to speed v in a time interval of Δt. A newer, more powerful sports car accelerates from 0 to $2v$ in the same time period. Assuming the energy coming from the engine appears only as kinetic energy of the cars, compare the power of the two cars.

27. A 3.50-kN piano is lifted by three workers at constant speed to an apartment 25.0 m above the street using a pulley system fastened to the roof of the building. Each worker is able to deliver 165 W of power, and the pulley system is 75.0% efficient (so that 25.0% of the mechanical energy is transformed to other forms due to friction in the pulley). Neglecting the mass of the pulley, find the time required to lift the piano from the street to the apartment.

28. A 650-kg elevator starts from rest. It moves upward for 3.00 s with constant acceleration until it reaches its cruising speed of 1.75 m/s. (a) What is the average power of the elevator motor during this time interval? (b) How does this power compare with the motor power when the elevator moves at its cruising speed?

29. A 4.00-kg particle moves along the x axis. Its position varies with time according to $x = t + 2.0t^3$, where x is in meters and t is in seconds. Find (a) the kinetic energy of the particle at any time t, (b) the acceleration of the particle and the force acting on it at time t, (c) the power being delivered to the particle at time t, and (d) the work done on the particle in the interval $t = 0$ to $t = 2.00$ s.

Topic 6. Dynamics of rotational motion.

Example 6.1. Fundamental equation of the dynamics of rotational motion

A block of mass m_1 = 100 g and a block of mass m_2 = 200 g are connected by a massless string over a pulley in the shape of a solid disk of mass m = 80 g (Figure 6.1). Find the acceleration the blocks acquire if left on their own? Friction is negligible.

Solution:

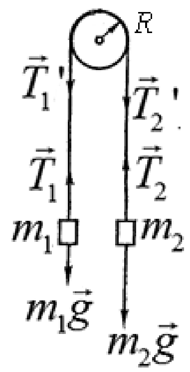


Figure 6.1.

To solve the problem we apply the fundamental laws of translational and rotational motion.
Two forces are acting on each of the moving blocks: the gravitational force $m\vec{g}$, directed downward, and the tension force $\vec{T}$, directed upward. Since $m_2 > m_1$, the acceleration vector $\vec{a}$ of the first block is directed upwards, while the acceleration vector of the second block is directed downwards.

Let's write the Newton's second law for the two blocks:

$$m_1\vec{g} + \vec{T}_1 = m_1\vec{a}; \qquad m_2\vec{g} + \vec{T}_2 = m_2\vec{a}.$$

Projections onto the vertical direction are:

$$-m_1 g + T_1 = m_1 a; \quad -m_2 g + T_2 = -m_2 a.$$

Then, the tension forces

$$T_1 = m_1 a + m_1 g; \qquad T_2 = m_2 g - m_2 a.$$

According to the fundamental equation of dynamics of rotational motion,

$M = J\beta$,

where M is the net torque exerted on the disk, J is its moment of inertia, β is its angular acceleration.

The magnitude of the net torque acting on the disk about its axis of rotation is

$M = T'_2R - T'_1R,$

where T'_2 and T'_1 are the forces exerted by the string on the rim of the disk. (The gravitational force exerted by the Earth on the disk and the normal force exerted by the axle on the disk both pass through the axis of rotation and therefore produce no torque.)

According to Newton's third law, the forces T'_2 and T'_1 are equal in magnitudes to the forces T_2 and T_1, respectively, but are opposite in directions. When the blocks move, the disk rotates clockwise. Consequently,

$M = (T_2 - T_1)R.$

The moment of inertia of the disk $J = mR^2/2$.

Because the blocks and disk are connected by a string that does not slip, the translational acceleration of the suspended blocks is equal to the tangential acceleration of a point on the disk's rim. Therefore, the angular acceleration β of the disk and the translational acceleration of the blocks are related by $\beta = a/R$.

Substituting these expressions into the fundamental law of the dynamics of rotational motion, we obtain:

$$(T_2 - T_1)R = \frac{mR^2}{2}\frac{a}{R}, \qquad T_2 - T_1 = \frac{ma}{2}.$$

Using the expressions for the tension forces found before,

$$m_2g - m_2a - (m_1g - m_1a) = \frac{ma}{2}; \qquad m_2g - m_1g = m_1a + m_2a + \frac{ma}{2}.$$

Eventually,

$$a = \frac{(m_2 - m_1)g}{m_1a + m_2a + ma/2}.$$

Calculations:

$$a = \frac{(0.2 - 0.1)}{0.2 + 0.1 + 0.04} \cdot 9.8 m/s^2 = 2.88 m/s^2.$$

Example 6.2. The law of conservation of the angular momentum

A platform in the form of a disk of radius R = 1.5 m and mass m_1 = 180 kg is rotating by inertia about a vertical axis with a frequency υ = 10 min^{-1}. A man of mass m_2 = 60 kg is standing in the center of the platform. What linear speed relative to the ground will the man have if he moves to the edge of the platform?

Solution:

According to the law of conservation of the angular momentum,

$$(J_1 + J_2)\omega = (J_1 + J_2')\omega'; \qquad \omega' = \frac{J_1 + J_2}{J_1 + J_2'}\omega,$$

where J_1 is the moment of inertia of the platform, J_2 is the moment of inertia of the man standing in the center of the platform, ω is the angular speed of the platform with the man standing in its center, J_2' is the moment of inertia of the man standing on the edge of the platform, ω' is the angular speed of the platform with the man standing on the edge.

The moment of inertia of the platform is equal to that of the disk, consequently, $J_1 = \frac{m_1 R^2}{2}$. The moment of inertia of the man is equal to that of the material point, therefore, $J_2 = 0$, $J_2' = m_2 R^2$. The angular speed of the platform before the man moves is equal to $\omega = 2\pi\upsilon$.

The linear speed of the man standing on the edge of the platform is related to the angular speed by the relation $v = \omega' R$.

Then, after substitution, we find the linear speed of the man:

$$v = \frac{J_1 + J_2}{J_1 + J_2'}\omega R = \frac{m_1 R^2/2}{m_1 R^2/2 + m_2 R^2} 2\pi\upsilon R = \frac{m_1}{m_1 + 2m_2} 2\pi\upsilon R.$$

The calculations:

$$v = \frac{180}{180 + 2 \cdot 60} 2\pi \frac{10}{60} 1.5 m/s = 0.942 m/s .$$

Example 6.3. Relative rotation.

A platform in the form of a disk can rotate about a vertical axis. A man of mass m_1 = 60 kg is standing at the edge of the platform. By what angle does the platform turn about the axis if the man walks along the edge of the platform and, bypassing it, returns to the initial point on the platform? The mass of the platform is m_2 =240 kg. The moment of inertia J of the man is considered as for a material point.

Solution:

Before the man starts walking, the man-platform system is at rest and its total angular momentum is zero. Then the walking man supplies an angular momentum to the platform and it starts rotating. According to the law of conservation of the angular momentum of the system,

$$0 = J_1 \omega_1 + J_2 \omega_2 ,$$

where J_1 is the moment of inertia of the man at the edge of the platform, for the material point $J_1 = m_1 R^2$;

J_2 is the moment of inertia of platform, for the disk $J_2 = \frac{m_2 R^2}{2}$;

ω_1 is the angular speed of the man relative to the ground;

ω_2 is the angular speed of the platform relative to the ground.

Let's find how the angular speeds of the platform and the man are related. Consider motion of the man relative to the reference frame associated with the platform. In that frame he passes complete circle along the edge of the platform. That is, he turns by the angle 2π relative to the platform. We can find the angular speed of the man in the "platform" reference frame as

$$\omega' = \frac{\Delta\varphi}{\Delta t} = \frac{2\pi}{\Delta t},$$

where Δt is the time of his motion.

As follows from the Galilean velocity addition law,

$$\omega_1 = \omega_2 - \frac{2\pi}{\Delta t},$$

where ω_1 is the man's angular speed relative to the ground, ω_2 is the angular speed of the "platform" frame relative to the ground, $\frac{2\pi}{\Delta t}$ is the man's angular speed relative to the moving "platform" frame; the negative sign indicates that the angular velocities of the man and the platform have opposite direction.

After substitution we obtain:

$$0 = m_1 R^2 \left(\omega_2 - \frac{2\pi}{\Delta t} \right) + \frac{m_2 R^2}{2} \omega_2,$$

and the angular speed of the platform is

$$\omega_2 = \frac{m_1 R^2 \left(2\pi / \Delta t \right)}{m_1 R^2 + m_2 R^2 / 2}$$

The time of the platform's motion is equal to the time of the man's motion along the edge, so the angular speed of the platform may be found as $\omega_2 = \frac{\Delta\varphi_2}{\Delta t}$. Consequently, the platform rotates by the angle

$$\Delta\varphi_2 = \frac{2\pi m_1 R^2}{R^2 (m_1 + m_2 / 2)} = \frac{2\pi m_1}{(m_1 + m_2 / 2)}.$$

Calculations:

$$\Delta\varphi_2 = \frac{2\pi \cdot 60}{(60 + 240 / 2)} \approx 2.1(\text{rad})$$

Example 6.4. Rolling motion.

A solid sphere of radius R and mass m is rolling without slipping after being released from rest at the top of the incline of angle θ and length l. Find the translational speed of the center of mass at the bottom of the incline and the magnitude of the translational acceleration of the center of mass.

Solution:

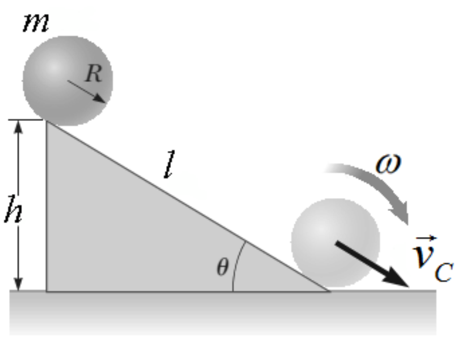


Figure 6.2.

Accelerated rolling motion is possible only if a friction force is present between the sphere and the incline to produce a net torque about the center of mass. Despite the presence of friction, no loss of mechanical energy occurs because the contact point is at rest relative to the surface at any instant. (On the other hand, if the sphere were to slip, mechanical energy of the sphere–incline–Earth system would decrease due to the nonconservative force of kinetic friction.)

Then, the law of conservation of the total mechanical energy can be used. For the sphere–Earth system, we define the zero configuration of gravitational potential energy to be when the sphere is at the bottom of the incline. Then at the top of the incline (see Figure 6.2)

$$U = mgh = mgl\sin\theta.$$

At the bottom, the gravitational potential energy is transformed into the kinetic energy of the rolling motion

$$K = \frac{I_C\omega^2}{2} + \frac{m{v_C}^2}{2},$$

where I_C is the moment of inertia of the sphere about the axis through its center of mass; $I_C = \frac{2mR^2}{5}$;

ω is the angular speed of rotation;

v_C is the translational speed of the center of mass of the sphere.

Therefore, conservation of mechanical energy gives

$$mgl\sin\theta = \frac{I_C\omega^2}{2} + \frac{mv_C{}^2}{2}.$$

Using the relation $v_C = \omega R$, we obtain

$$mgl\sin\theta = \frac{I_C v_C{}^2}{2R^2} + \frac{mv_C{}^2}{2};$$

$$v_C = \sqrt{\frac{mgl\sin\theta}{I_C/2R^2 + m/2}}.$$

Substituting the value for I_C we obtain

$$v_C = \sqrt{\frac{mgl\sin\theta}{m/5 + m/2}} = \sqrt{\frac{10}{7}gl\sin\theta}.$$

To calculate the translational acceleration of the center of mass, notice that it passes the distance l along the incline at constant acceleration. For an object starting from rest and moving through a distance l under constant acceleration:

$$l = \frac{at^2}{2} = \frac{v^2}{2a},$$

and

$$a = \frac{v^2}{2l} = \frac{1}{2l}\cdot\frac{10}{7}gl\sin\theta = \frac{5}{7}g\sin\theta.$$

We have found $v_C = \sqrt{\frac{10}{7}gl\sin\theta}$ and $a = \frac{5}{7}g\sin\theta$.

Problems

1. A rotating wheel requires 3.00 s to rotate through 37.0 revolutions. Its angular speed at the end of the 3.00-s interval is 98.0 rad/s. What is the constant angular acceleration of the wheel?
2. A wheel starts from rest and rotates with constant angular acceleration to reach an angular speed of 12.0 rad/s in 3.00 s. Find (a) the magnitude of the angular acceleration of the wheel and (b) the angle in radians through which it rotates in this time interval.
3. A centrifuge in a medical laboratory rotates at an angular speed of 3 600 rev/min. When switched off, it rotates through 50.0 revolutions before coming to rest. Find the constant angular acceleration of the centrifuge.
4. During a certain time interval, the angular position of a swinging door is described by $\theta = 5.00 + 10.0t + 2.00t^2$, where uis in radians and tis in seconds. Determine the angular position, angular speed, and angular acceleration of the door (a) at $t = 0$ and (b) at $t = 3.00$ s.
5. A bar on a hinge starts from rest and rotates with an angular acceleration $\beta = 10 + 6t$, where β is in rad/s^2 and t is in seconds. Determine the angle in radians through which the bar turns in the first 4.00 s.
6. A disk 8.00 cm in radius rotates at a constant rate of 1200 rev/min about its central axis. Determine (a) its angular speed in radians per second, (b) the tangential speed at a point 3.00 cm from its center, (c) the radial acceleration of a point on the rim, and (d) the total distance a point on the rim moves in 2.00 s.
7. The four particles in Figure 6.3 are connected by rigid rods of negligible mass. The origin is at the center of the rectangle. The system rotates in the *xy* plane about the *z* axis with an angular speed of 6.00 rad/s. Calculate (a) the moment of inertia of the system about the *z* axis and (b) the rotational kinetic energy of the system.

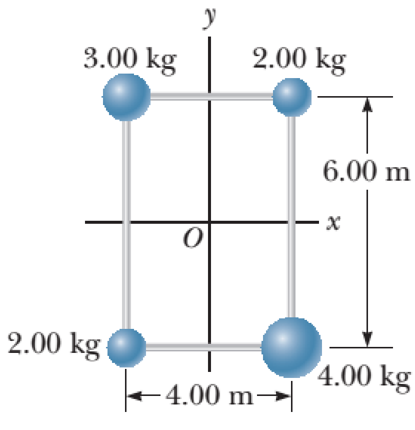


Figure 6.3.

8. A uniform, thin, solid door has height 2.20 m, width 0.870 m, and mass 23.0 kg. (a) Find its moment of inertia for rotation on its hinges. (b) Is any piece of data unnecessary?

9. In Figure 6.4, the cam is a circular disk of radius R with a hole of diameter R cut through it. As shown in the figure, the hole does not pass through the center of the disk.

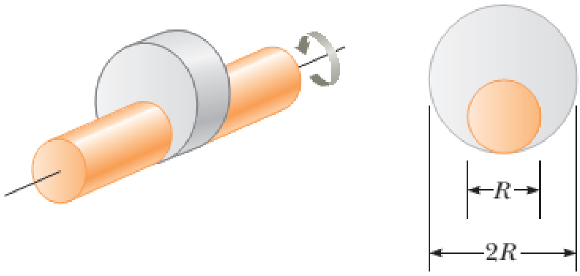


Figure 6.4.

10. The cam with the hole cut out has mass M. The cam is mounted on a uniform, solid, cylindrical shaft of diameter R and also of mass M. What is the kinetic energy of the cam–shaft combination when it is rotating with angular speed ω about the shaft's axis?

11. Find the net torque on the wheel in Figure 6.5 about the axle through O, taking a = 10.0 cm and b = 25.0 cm.

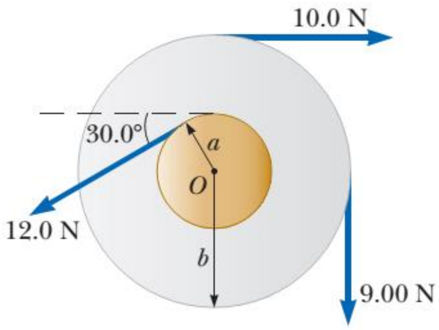


Figure 6.5.

12. A grinding wheel is in the form of a uniform solid disk of radius 7.00 cm and mass 2.00 kg. It starts from rest and accelerates uniformly under the action of the constant torque of 0.600 N ?m that the motor exerts on the wheel. (a) How long does the wheel take to reach its final operating speed of 1 200 rev/min? (b) Through how many revolutions does it turn while accelerating?

13. A block of mass $m_1 = 2.00$ kg and a block of mass $m_2 = 6.00$ kg are connected by a massless string over a pulley in the shape of a solid disk having radius $R = 0.250$ m and mass $M = 10.0$ kg. The fixed, wedge-shaped ramp makes an angle of $\theta = 30.0^0$ as shown in Figure 6.6.

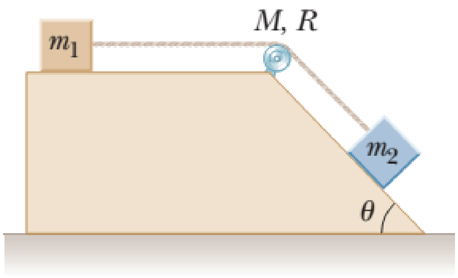


Figure 6.6.

The coefficient of kinetic friction is 0.360 for both blocks. (a) Draw force diagrams of both blocks and of the pulley. Determine (b) the acceleration of the two blocks and (c) the tensions in the string on both sides of the pulley.

14. A potter's wheel—a thick stone disk of radius 0.500 m and mass 100 kg—is freely rotating at 50.0 rev/min. The potter can stop the wheel in 6.00 s by pressing a wet rag against the rim and exerting a radially inward force of 70.0 N. Find the effective coefficient of kinetic friction between wheel and rag.

15. Consider the system shown in Figure 6.7 with $m_1 = 20.0$ kg, $m_2 = 12.5$ kg, $R = 0.200$ m, and the mass of the pulley $M = 5.00$ kg. Object m_2 is resting on the floor, and object m_1 is 4.00 m above the floor when it is released from rest. The pulley axis is frictionless. The cord is light, does not stretch, and does not slip on the pulley. (a) Calculate the time interval required for m_1 to hit the floor. (b) How would your answer change if the pulley were massless?

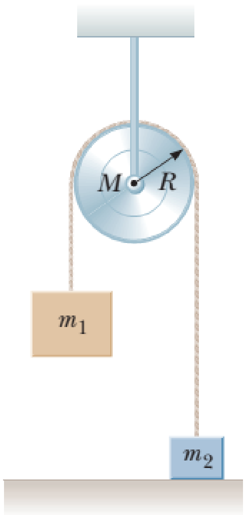


Figure 6.7.

16. A horizontal 800-N merry-go-round is a solid disk of radius 1.50 m and is started from rest by a constant horizontal force of 50.0 N applied tangentially to the edge of the disk. Find the kinetic energy of the disk after 3.00 s.

17. In Figure 6.8, the hanging object has a mass of m_1 = 0.420 kg; the sliding block has a mass of m_2 = 0.850 kg; and the pulley is a hollow cylinder with a mass of M = 0.350 kg, an inner radius of R_1 = 0.020 0 m, and an outer radius of R_2 = 0.030 0 m. Assume the mass of the spokes is negligible.

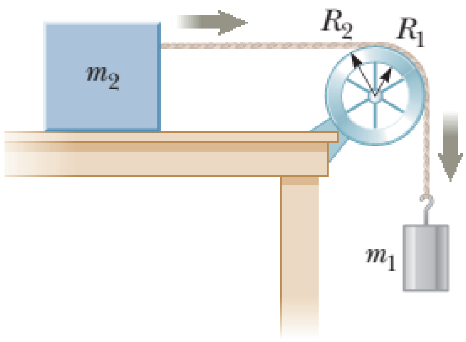


Figure 6.8.

The coefficient of kinetic friction between the block and the horizontal surface is μ = 0.250. The pulley turns without friction on its axle. The light cord does not stretch and does not slip on the pulley. The block has a velocity of v_i = 0.820 m/s toward the pulley when it passes a reference point on the table. (a) Find the acceleration of the block. (b) Use energy methods to predict its speed after it has moved to a second point, 0.700 m away.

18. An object with a mass of m = 5.10 kg is attached to the free end of a light string wrapped around a reel of radius R = 0.250 m and mass M = 3.00 kg. The reel is a solid disk, free to rotate in a vertical plane about the horizontal axis passing through its center as shown in Figure 6.9. The suspended object is released from rest 6.00 m above the floor. Determine (a) the tension in the string, (b) the acceleration of the object, and (c) the speed with which the object hits the floor. (d) Verify your answer to part (c) by using the isolated system (energy) model.

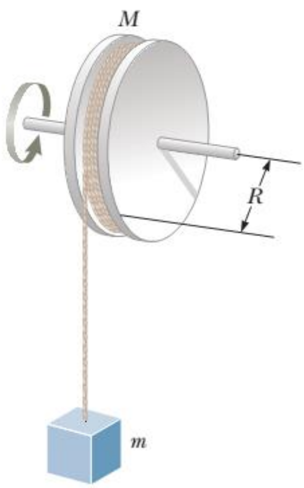


Figure 6.9.

19. A uniform solid disk of radius R and mass M is free to rotate on a frictionless pivot through a point on its rim (Fig. 6.10). If the disk is released from rest in the position shown by the copper-colored circle, (a) what is the speed of its center of mass when the disk reaches the position indicated by the dashed

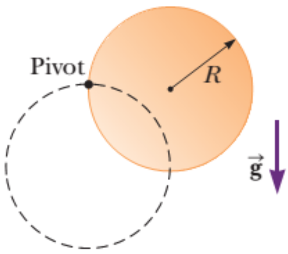


Figure 6.10.

circle? (b) What is the speed of the lowest point on the disk in the dashed position?

20. A cylinder of mass 10.0 kg rolls without slipping on a horizontal surface. At a certain instant, its center of mass has a speed of 10.0 m/s. Determine (a) the translational kinetic energy of its center of mass, (b) the rotational kinetic energy about its center of mass, and (c) its total energy.

21. Determine the acceleration of the center of mass of a uniform solid disk rolling down an incline making angle θ with the horizontal. Compare the found acceleration with that of a uniform hoop.

22. A uniform solid disk and a uniform hoop are placed side by side at the top of an incline of height *h*. (a) If they are released from rest and roll without slipping, which object reaches the bottom first? (b) Verify your answer by calculating their speeds when they reach the bottom in terms of *h*.

23. A clown balances a small spherical grape at the top of his bald head, which also has the shape of a sphere. After drawing sufficient applause, the grape starts from rest and rolls down without slipping. It will leave contact with the clown's scalp when the radial line joining it to the center of curvature makes what angle with the vertical?

24. A conical pendulum consists of a bob of mass *m* in motion in a circular path in a horizontal plane as shown in Figure 6.11. During the motion, the supporting wire of length ,maintains a constant angle θ with the vertical. Find the magnitude of the angular momentum of the bob about the vertical dashed line.

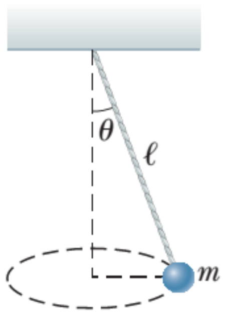
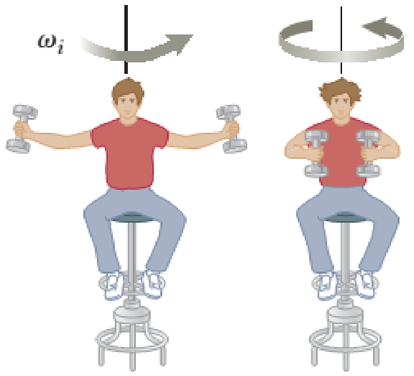


Figure 6.11.

25. A uniform solid disk of mass m = 3.00 kg and radius r = 0.200 m rotates about a fixed axis perpendicular to its face with angular frequency 6.00 rad/s. Calculate the magnitude of the angular momentum of the disk when the axis of rotation (a) passes through its center of mass and (b) passes through a point midway between the center and the rim.

26. A playground merry-go-round of radius R = 2.00 m has a moment of inertia I = 250 kg·m^2 and is rotating at 10.0 rev/min about a frictionless, vertical axle. Facing the axle, a 25.0-kg child hops onto the merry-go-round and manages to sit down on the edge. What is the new angular speed of the merry-go-round?

27. A student sits on a freely rotating stool holding two dumbbells, each of mass 3.00 kg (Fig. 6.12). When his arms are extended horizontally (Fig. 6.12a), the dumbbells are 1.00 m from the axis of rotation and the student rotates with an angular speed of 0.750 rad/s. The moment of inertia of the student plus stool is 3.00 kg·m^2 and is assumed to be constant.



(a) (b)

Figure 6.12.

The student pulls the dumbbells inward horizontally to a position 0.300 m from the rotation axis (Fig. 6.12b). (a) Find the new angular speed of the student. (b) Find the kinetic energy of the rotating system before and after he pulls the dumbbells inward.

28. A wooden block of mass M resting on a frictionless, horizontal surface is attached to a rigid rod of length l, and of negligible mass (Fig. 6.13). The rod is pivoted at the other end. A bullet of mass m traveling parallel to the horizontal surface and perpendicular to the rod with speed v hits the block and becomes embedded in it.

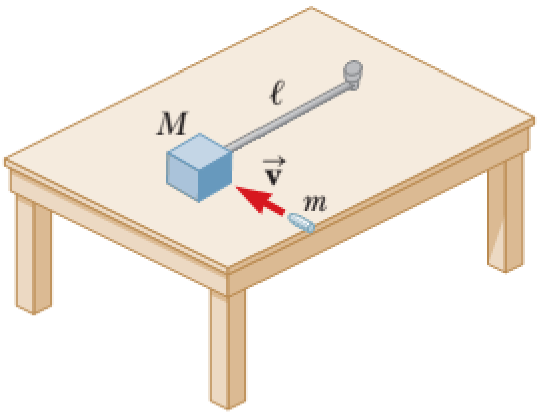


Figure 6.13.

(a) What is the angular momentum of the bullet–block system about a vertical axis through the pivot? (b) What fraction of the original kinetic energy of the bullet is converted into internal energy in the system during the collision?

29. A 0.005 00-kg bullet traveling horizontally with speed 10^3 m/s strikes an 18.0-kg door, imbedding itself 10.0 cm from the side opposite the hinges as shown in Figure 6.14. The 1.00-m wide door is free to swing on its frictionless hinges. At what angular speed does the door swing open immediately after the collision?

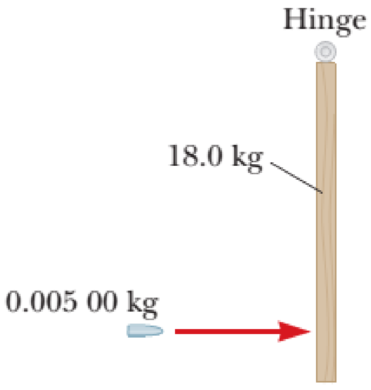


Figure 6.14.

MOLECULAR PHYSICS AND THERMODYNAMICS

Topic 7. Kinetic theory of gases.

Example 7.1. Microscopic characteristics of the ideal gas

Find the average kinetic energy per one molecule of ammonia NH_3 at a temperature of t = 27°C and the average energy of rotational motion of this molecule at the same temperature.

Solution:

The average total kinetic energy per molecule is

$$\bar{E} = \frac{i}{2} k_B T ,$$

where i is the number of degrees of freedom of the molecule; k_B is the Boltzmann constant; T is the thermodynamic temperature of the gas: $T = t^0 + T_0$, where T_0 = 273 K. The number of degrees of freedom i for a four-atomic ammonia molecule equals 6.

Substituting numerical values, we obtain:

$$\bar{E} = \frac{6}{2} 1.38 \cdot 10^{-23} (27 + 273) = 1.242 \cdot 10^{-20} \text{ (J)}.$$

The average energy of rotational motion per molecule is determined by the formula

$$\bar{E}_{rot} = \frac{i-3}{2} k_B T ,$$

where the subtracted number 3 means the number of translational degrees of freedom per molecule.

Substituting numerical values:

$$\bar{E}_{rot} = \frac{6-3}{2} 1.38 \cdot 10^{-23} (27+273) = 6.21 \cdot 10^{-21} \text{ (J)}.$$

Example 7.2. Microscopic characteristics of the ideal gas

The mean free path $<l>$ of a carbon dioxide molecule under standard conditions is 40 nm. Determine the average arithmetic speed v_{avg} of the molecules and the number of collisions z experienced by the molecule in 1 second.

Solution:

The average arithmetic speed of the molecules is given by the formula

$$v_{avg} = \sqrt{\frac{8RT}{\pi M}} \ ,$$

here M is the molar mass of the substance.

Substituting the numerical values, we obtain

v_{avg} = 362 m/s.

The mean free path is given by the relation

$$< l > = \frac{< v > t}{< z > t},$$

then the mean number of collisions in unit time is

$$< z > = \frac{v_{avg}}{< l >},$$

and substituting the numerical values v_{avg} = 362 m/s, $<l>$ = 40 nm = $4 \cdot 10^{-8}$ m, we obtain

$<z> = 9.05 \cdot 10^9$ s^{-1}.

Example 7.3. The Boltzmann distribution law

Dust particles of mass $m = 10^{-18}$ g are suspended in the air. Determine the thickness of the air layer, within which there is no more than 1% difference in concentration of the dust particles. The temperature T of the air within the entire volume is the same and equals to 300 K.

Solution:

In the state of thermodynamic equilibrium, concentration of the dust particles depends only on the z coordinate along the vertical axis. In that case, the concentration can be found from the Boltzmann distribution law:

$$n = n(0)e^{-\frac{U(z)}{k_B T}},$$

where $U(z)$ is the potential energy. In the homogeneous gravitational field $U = mgz$, then

$$n = n(0)e^{-\frac{mgz}{k_B T}}.$$

According to the formulation of the problem, the change in concentration Δn with height is small comparing to n ($\Delta n/n = 0.01$). Therefore, without a significant error, we can consider the change in concentration Δn as the differential dn.

Differentiating the expression for n with respect to z, we obtain

$$dn = -\frac{mg}{k_B T} n(0) e^{-\frac{mgz}{k_B T}} dz = -\frac{mg}{k_B T} n dz .$$

Then, the change in the coordinate

$$dz = -\frac{k_B T}{mg}\frac{dn}{n}$$

The negative sign indicates that the positive change in the coordinate ($dz > 0$) corresponds to the decrease in the relative concentration ($dn < 0$). We omit the negative sign (in this case it is not essential) and replace the differentials dz and dn by the finite increments Δz and Δn:

$$\Delta z = \frac{k_B T}{mg}\frac{\Delta n}{n}.$$

By substituting the values $\Delta n/n = 0.01$, $k_B = 1.38 \cdot 10^{-23}$ J/K, $T = 300$ K, $m = 10^{-21}$ kg, $g = 9.81$ m/s^2 and making calculations, we obtain

$\Delta z = 4.23$ mm.

As can be seen from the obtained result, concentration of even such small dust particles ($m = 10^{-18}$ g) changes very rapidly with height.

Example 7.4. Barometric formula

A barometer in a cockpit of a flying airplane always shows the same pressure $p = 79$ kPa, so a pilot considers an altitude h of the flight to be unchanged. However, the air temperature outside the plane changes from $t^0 = 5$ °C to $t^0 = 1$ °C. What error Δh in the definition of altitude is made by the pilot? The pressure p_0 at the Earth's surface is assumed to be standard.

Solution:

Consider the barometric formula

$$p = p_0 e^{-\frac{Mg}{RT}h},$$

where M is the molar mass of the air. The barometer can show unchanging pressure p at different temperatures T_1 and T_2 only if the airplane is not at the same altitude h (which is considered unchanged by the pilot), but at some other altitude h_2.

Let us use the barometric formula for these two cases:

$$p = p_0 e^{-\frac{Mg}{RT_1}h_1}\ ;\ .\ p = p_0 e^{-\frac{Mg}{RT_2}h_2}$$

We can express the altitudes h_1 and h_2:

$$h_1 = \frac{RT_1}{Mg}\ln\left(\frac{p_0}{p}\right); \qquad h_2 = \frac{RT_2}{Mg}\ln\left(\frac{p_0}{p}\right).$$

Now the difference Δh:

$$\Delta h = h_2 - h_1 = \frac{R\ln\left(p_0/p\right)}{Mg}\left(T_2 - T_1\right) \ .$$

Substituting the numerical values:

$$\Delta h = \frac{8.31\cdot\ln\left(101/79\right)}{29\cdot 10^{-3}\cdot 9.8}\left(1-5\right) = -28.5(\text{m})$$

The negative sign means that $h_2 < h_1$ and, consequently, that the airplane has descended by 28.5 m comparing to the assumed altitude.

Example 7.5. The equation of state for an ideal gas

A 10-L cylinder contains helium at a pressure of p_1 = l MPa at a temperature of T_1 = 300 K. After 10 g of helium is used from the balloon, the temperature in the cylinder droppes to T_2 = 290 K. Determine the pressure p_2 of the remaining helium in the cylinder.

Solution:

To solve the problem, we use the equation of state for an ideal gas. For the initial state:

$$p_1 V = \frac{m_1}{M} RT_1,$$

and for the final state:

$$p_2 V = \frac{m_2}{M} RT_2,$$

where m_1 and m_2 are the masses of helium in the initial and final states. Solving for m_1 and m_2:

$$m_1 = \frac{p_1 VM}{RT_1}; \qquad m_2 = \frac{p_2 VM}{RT_2}.$$

Then

$$\Delta m = \frac{VM}{R}\left(\frac{p_1}{T_1} - \frac{p_2}{T_2}\right),$$

and the pressure in the final state:

$$p_2 = \frac{p_1 T_2}{T_1} - \frac{\Delta m R T_2}{MV} .$$

According to the Mendeleev's periodic table of elements, the molar mass of helium is $M = 4 \cdot 10^{-3}$ kg/mol. Substituting the numerical values, we obtain

$$p_2 = \frac{10^6 \cdot 290}{300_1} - \frac{10 \cdot 10^{-3} \cdot 8.31 \cdot 290}{4 \cdot 10^{-3} \cdot 10 \cdot 10^{-3}} = 3.64 \cdot 10^5 \text{ (Pa)}$$

Problems

1. A cylinder contains a mixture of helium and argon gas in equilibrium at 150^0C. (a) What is the average kinetic energy for each type of gas molecule? (b) What is the rms speed of each type of molecule?
2. A 2.00-mol sample of oxygen gas is confined to a 5.00-L vessel at a pressure of 8.00 atm. Find the average translational kinetic energy of the oxygen molecules under these conditions.
3. Oxygen, modeled as an ideal gas, is in a container and has a temperature of 77.0^0C. What is the rms-average magnitude of the momentum of the gas molecules in the container?
4. In a period of 1.00 s, $5.00 \cdot 10^{23}$ nitrogen molecules strike a wall with an area of 8.00 cm^2. Assume the molecules move with a speed of 300 m/s and strike the wall head-

on in elastic collisions. What is the pressure exerted on the wall? Note: the mass of one N_2 molecule is $4.65 \cdot 10^{-26}$ kg.

5. In a 30.0-s interval, 500 hailstones strike a glass window of area 0.600 m^2 at an angle of 45.0^0 to the window surface. Each hailstone has a mass of 5.00 g and a speed of 8.00 m/s. Assuming the collisions are elastic, find (a) the average force and (b) the average pressure on the window during this interval.
6. If the diameter of an oxygen molecule is $2.00 \cdot 10^{-10}$ m, find the mean free path of the molecules in a scuba tank that has a volume of 12.0 L and is filled with oxygen at a gauge pressure of 100 atm at a temperature of 25.0^0C. What is the average time interval between molecular collisions for a molecule of this gas?
7. Gas is confined in a tank at a pressure of 11.0 atm and a temperature of 25.0^0C. If two-thirds of the gas is withdrawn and the temperature is raised to 75.0^0C, what is the pressure of the gas remaining in the tank?
8. A rigid tank contains 1.50 moles of an ideal gas. Determine the number of moles of gas that must be withdrawn from the tank to lower the pressure of the gas from 25.0 atm to 5.00 atm. Assume the volume of the tank and the temperature of the gas remain constant during this operation.
9. Gas is contained in an 8.00-L vessel at a temperature of 20.0^0C and a pressure of 9.00 atm. (a) Determine the number of moles of gas in the vessel. (b) How many molecules are in the vessel?
10. An auditorium has dimensions 10.0 m × 320.0 m × 330.0 m. How many molecules of air fill the auditorium at 20.0^0C and a pressure of 101 kPa (1.00 atm)?
11. An automobile tire is inflated with air originally at 10.0^0C and normal atmospheric pressure. During the process, the air is compressed to 28.0% of its original volume and the temperature is increased to 40.0^0C. (a) What is the tire pressure? (b) After the car is driven at high speed, the tire's air temperature rises to 85.0^0C and the tire's interior volume increases by 2.00%. What is the new tire pressure (absolute)?
12. The mass of a hot-air balloon and its cargo (not including the air inside) is 200 kg. The air outside is at 10.0^0C and 101 kPa. The volume of the balloon is 400 m^3.

To what temperature must the air in the balloon be warmed before the balloon will lift off? (Air density at 10.0^0C is 1.244 kg/m^3.)

13. At 25.0 m below the surface of the sea, where the temperature is 5.00^0C, a diver exhales an air bubble having a volume of 1.00 cm^3. If the surface temperature of the sea is 20.0^0C, what is the volume of the bubble just before it breaks the surface?

14. A bicycle tire is inflated to a gauge pressure of 2.50 atm when the temperature is 15.0^0C. While a man rides the bicycle, the temperature of the tire rises to 45.0^0C. Assuming the volume of the tire does not change, find the gauge pressure in the tire at the higher temperature.

Topic 8. The first law of thermodynamics.

Example 8.1. Heat. Internal energy. Work.

What amount of energy by heat is absorbed by the hydrogen of mass m = 0.2 kg when it is heated from the temperature $t^0{}_1$ = 0 °C to the temperature $t^0{}_2$ = 100 °C at constant pressure? Find also the change in the internal energy of the gas and the work done by the gas.

Solution:

The heat Q absorbed by the gas in isobaric process can determined by the formula

$$Q = \upsilon C_p \Delta T ,$$

where $\upsilon = \dfrac{m}{M}$ is the amount of substance, $C_p = \dfrac{i+2}{2}R$ is molar heat capacity at constant pressure; ΔT is the change in the gas temperature. So,

$$Q = \frac{m}{M}\frac{i+2}{2}R\Delta T .$$

Substituting the numerical values, we obtain

$$Q = \frac{0.2}{2\cdot 10^{-3}}\frac{5+2}{2}\cdot 8.31\cdot 100 = 291\,(\text{kJ}).$$

The change in the internal energy is expressed by the formula

$$\Delta U = \frac{m}{M}\frac{i}{2}R\Delta T ,$$

then

$$\Delta U = \frac{0.2}{2\cdot 10^{-3}}\frac{5}{2}\cdot 8.31\cdot 100 = 208\,(\text{kJ}).$$

The work done by the expanding gas can be expressed from the first law of thermodynamics:

$$Q = \Delta U + A_{by\_gas},$$

and

$$A_{by\_gas} = Q - \Delta U .$$

Substituting the numerical values of Q and ΔU, we find

A = 83 kJ.

Example 8.2. Heat capacity at constant pressure and at constant volume

Nitrogen is heated at constant pressure and Q = 21 kJ of energy is supplied to the gas by heat. Find the work done by the gas and the change in its internal energy.

Solution:

According to the first law of thermodynamics,

$$Q = \Delta U + A_{by\_gas},$$

where ΔU is the change in the internal energy and A_{by_gas} is the work done by the expanding gas.

The heat Q absorbed by the gas in isobaric equals

$$Q = \upsilon C_p \Delta T ,$$

where υ is the amount of substance, $C_p = \frac{i+2}{2} R$ is molar heat capacity at constant pressure; ΔT is the change in the gas temperature.

The change in the internal energy by definition can be found as

$$\Delta U = \upsilon \frac{i}{2} R \Delta T = \upsilon C_V \Delta T ,$$

where υ is the amount of substance, $C_V = \frac{i}{2}R$ is molar heat capacity at constant volume; ΔT is the change in the gas temperature.

By taking ratio of the two last expressions, we can find that

$$\frac{\Delta U}{Q} = \frac{C_V}{C_p} = \frac{i}{i+2}.$$

Then, the change in the internal energy equals

$$\Delta U = \frac{i}{i+2}Q.$$

Substituting the numerical values, we find

$$\Delta U = \frac{5}{7} \cdot 21 \cdot 10^3 = 15 \cdot 10^3 \text{ (J)}$$

From the first law of thermodynamics, the work done by the gas is

$$A_{by\_gas} = Q - \Delta U = Q - \frac{i}{i+2}Q = \frac{2}{i+2}Q.$$

Substituting the numerical values, we find

$$A = \frac{2}{7} \cdot 21 \cdot 10^3 = 6 \cdot 10^3 \text{ (J)}$$

Example 8.3. Heat capacity at a quasi-static process

An ideal gas, whose molar heat capacity at constant volume C_V is known, is taken through a quasi-static process described by 1) $T = T_0 e^{\alpha V}$; 2) $p = p_0 e^{\alpha V}$, where T_0, p_0, α are constants. Find the molar heat capacity of the gas as the function of its volume.

Solution:

According to the first law of thermodynamics,

$$\delta Q = dU + \delta A,$$

where dU is the change in the internal energy and δA is the work done by the expanding gas.

By substituting the expressions for heat $\delta Q = \upsilon CdT$, internal energy $dU = \frac{i}{2}\upsilon RdT = \upsilon C_V dT$ and work $\delta A = pdV$ into the first law of thermodynamics, we obtain

$$\upsilon CdT = \upsilon C_V dT + pdV;$$

$$C = C_V + \frac{pdV}{\upsilon dT}.$$

1. As $T = T_0 e^{\alpha V}$, we can find the derivative $\frac{dT}{dV} = T_0 \alpha e^{\alpha V}$. Then,

$$\frac{dV}{dT} = \frac{1}{T_0 \alpha e^{\alpha V}} = \frac{1}{\alpha T}, \text{ and}$$

$$C = C_V + \frac{p}{\upsilon \alpha T}.$$

According to the equation of state for an ideal gas, $pV = \upsilon RT$; $p = \frac{\upsilon RT}{V}$, and $C = C_V + \frac{\upsilon RT}{\upsilon \alpha TV}$. So,

$$C = C_V + \frac{R}{\alpha V}.$$

2. As $p = p_0 e^{\alpha V}$, then according to the equation of state for an ideal gas, $V = \frac{\upsilon RT}{p} = \frac{\upsilon RT}{p_0 e^{\alpha V}}$, or $Ve^{\alpha V} = \frac{\upsilon R}{p_0} T$. We can differentiate the both sides of this expression:

$$d\left(Ve^{\alpha V}\right) = d\left(\frac{\upsilon R}{p_0} T\right);$$

$$\left(e^{\alpha V}+V\alpha e^{\alpha V}\right)dV=\frac{\upsilon R}{p_0}dT;$$

$$\frac{dV}{dT}=\frac{\upsilon R}{p_0\left(e^{\alpha V}+V\alpha e^{\alpha V}\right)}=\frac{\upsilon R}{p(1+V\alpha)}.$$

By substituting this expression into the equation for the molar heat capacity, we obtain $C=C_V+\frac{p}{\upsilon}\cdot\frac{\upsilon R}{p(1+V\alpha)}$, and

$$C=C_V+\frac{R}{1+V\alpha}.$$

Example 8.4. Thermodynamic processes

Oxygen occupies a volume of V_1 = 1 m^3 and is under a pressure of p_1 = 200 kPa. The gas was first heated at a constant pressure to a volume of V_2 = 3 m^3, and then at constant volume up to a pressure of p_2 = 500 kPa. Build the PV-diagram of the process and find: 1) the change in the internal energy of the gas; 2) the work done by the gas; 3) the amount of energy by heat transferred to the gas.

Solution:

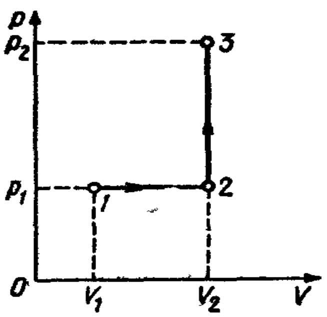


Figure 8.1.

Let's build the PV-diagram of the process (Figure 8.1). On the diagram the points 1, 2, 3 indicate the gas states characterized by the parameters (p_1, V_1, T_1), (p_1, V_2, T_2), (p_2, V_2, T_3).

1. The change in the internal energy of the gas during its transition from the state 1 to the state 3 is expressed by the formula

$$\Delta U=\frac{m}{M}\frac{i}{2}R\Delta T=\frac{m}{M}\frac{i}{2}R\left(T_3-T_1\right),$$

where M is the molar mass of the oxygen. The temperatures T_1 and T_3 can be found from the equation of state for the ideal gas:

$$T_1 = \frac{Mp_1V_1}{mR}; \qquad T_3 = \frac{Mp_2V_2}{mR}$$

Taking this into account,

$$\Delta U = \frac{i}{2}\left(p_2V_2 - p_1V_1\right)$$

Substituting the numerical values (considering that oxygen is a diatomic gas and $i = 5$) we obtain:

$\Delta U = 3.25$ MJ.

2. The total work done by the gas is equal to

$A = A_1 + A_2$,

where A_1 is work done during the process 1-2; A_2 is work done during the process 2-3. As we know, the work done by the gas is

$$\delta A = pdV.$$

The process 1-2 occurs at constant pressure (p = const). In that case the work is expressed by the formula

$A_1 = p_1\Delta V = p_1(V_2 - V_1)$.

The process 2-3 occurs at constant volume. That is the volume of the gas does not change, $dV = 0$, and, therefore, the work done by the gas is zero ($A_2 = 0$). So,

$A = A_1 = p_1(V_2 - V_1)$.

Substituting the numerical values we obtain:

$A = 0.4$ MJ

3. According to the first law of thermodynamics, the amount of energy transferred to the gas by heat Q is equal to the sum of the work A done by the gas and the change in the gas internal energy ΔU:

$Q = A + \Delta U$, or Q = 3.65 MJ.

Example 8.5. Thermodynamic processes

Find the equation of a thermodynamic process (in variables T, V) if the molar heat capacity of a gas is changing according to the law $C = C_V + \alpha T$, where α is a constant.

Solution:

According to the first law of thermodynamics,

$$\delta Q = dU + \delta A,$$

where $\delta Q = \upsilon C dT$ is the elementary amount of heat, $dU = \upsilon C_V dT$ is the change in the internal energy and $\delta A = pdV$ is the elementary amount of work done by the expanding gas.

Then, we obtain the expression for the molar heat capacity

$$C = C_V + \frac{pdV}{\upsilon dT}.$$

Comparing it with the expression given in the formulation of the problem, we see that

$$\frac{pdV}{\upsilon dT} = \alpha T.$$

According to the equation of state for an ideal gas, $p = \frac{\upsilon RT}{V}$, and we obtain

$$\frac{RTdV}{VdT} = \alpha T; \qquad \frac{RdV}{VdT} = \alpha; \qquad \frac{dV}{V} = \frac{\alpha}{R} dT.$$

Now we can take integral of the both sides of this expression and obtain

$$\ln \frac{V}{V_0} = \frac{\alpha}{R} T ,$$

where V_0 is the initial volume occupied by the gas. Then,

$$V = V_0 e^{\frac{\alpha}{R} T} .$$

Problems

1. What mass of water at 25.0°C must be allowed to come to thermal equilibrium with a 1.85-kg cube of aluminum initially at 150°C to lower the temperature of the aluminum to 65.0°C? Assume any water turned to steam subsequently condenses.
2. How long would it take a 1 000 W heater to melt 1.00 kg of ice at -20.0°C, assuming all the energy from the heater is absorbed by the ice?
3. A 3.00-g copper coin at 25.0°C drops 50.0 m to the ground. (a) Assuming 60.0% of the change in gravitational potential energy of the coin– Earth system goes into increasing the internal energy of the coin, determine the coin's final temperature.
4. How much energy is required to change a 40.0-g ice cube from ice at -10.0°C to steam at 110°C?
5. A 3.00-g lead bullet at 30.0°C is fired at a speed of 240 m/s into a large block of ice at 0°C, in which it becomes embedded. What quantity of ice melts?
6. In an insulated vessel, 250 g of ice at 0°C is added to 600 g of water at 18.0°C. (a) What is the final temperature of the system? (b) How much ice remains when the system reaches equilibrium?
7. An ideal gas is enclosed in a cylinder with a movable piston on top of it. The piston has a mass of 8000 g and an area of 5.00 cm^2 and is free to slide up and down, keeping the pressure of the gas constant. How much work is done on the gas as the temperature of 0.200 mol of the gas is raised from 20.0°C to 300°C?

8. An ideal gas is taken through a quasi-static process described by $p = \alpha V^2$, with $\alpha = 5.00$ atm/m^6. The gas is expanded to twice its original volume of 1.00 m^3. How much work is done on the expanding gas in this process?

9. An ideal gas initially at 300 K undergoes an isobaric expansion at 2.50 kPa. If the volume increases from 1.00 m^3 to 3.00 m^3 and 12.5 kJ is transferred to the gas by heat, what are (a) the change in its internal energy and (b) its final temperature?

10. One mole of an ideal gas does 3 000 J of work on its surroundings as it expands isothermally to a final pressure of 1.00 atm and volume of 25.0 L. Determine (a) the initial volume and (b) the temperature of the gas.

11. A sample of a diatomic ideal gas has pressure p and volume V. When the gas is warmed, its pressure triples and its volume doubles. This warming process includes two steps, the first at constant pressure and the second at constant volume. Determine the amount of energy transferred to the gas by heat.

12. In a (a) constant-volume process and (b) constant-pressure process, 209 J of energy is transferred by heat to 1.00 mol of an ideal monatomic gas initially at 300 K. Find the work done on the gas, the increase in internal energy of the gas, and its final temperature.

13. During the compression stroke of a certain gasoline engine, the pressure increases from 1.00 atm to 20.0 atm. If the process is adiabatic and the air–fuel mixture behaves as a diatomic ideal gas, (a) by what factor does the volume change and (b) by what factor does the temperature change? Assuming the compression starts with 0.016 0 mol of gas at 27.0^{0}C, find the values of (c) Q, (d) ΔU_{int}, and (e) A that characterize the process.

14. Air in a thundercloud expands as it rises. If its initial temperature is 300 K and no energy is lost by thermal conduction on expansion, what is its temperature when the initial volume has doubled?

15. How much work is required to compress 5.00 mol of air at 20.0^{0}C and 1.00 atm to one-tenth of the original volume (a) by an isothermal process? (b) How much work is required to produce the same compression in an adiabatic process? (c) What is the final pressure in part (a)? (d) What is the final pressure in part (b)?

16. As a 1.00-mol sample of a monatomic ideal gas expands adiabatically, the work done on it is $22.50 \cdot 10^3$ J. The initial temperature and pressure of the gas are 500 K and 3.60 atm. Calculate (a) the final temperature and (b) the final pressure.

17. A sample of a monatomic ideal gas occupies 5.00 L at atmospheric pressure and 300 K (point A in Figure 8.2). It is warmed at constant volume to 3.00 atm (point B). Then it is allowed to expand isothermally to 1.00 atm (point C) and at last compressed isobarically to its original state. (a) Find the number of moles in the sample. Find (b) the temperature at point B, (c) the temperature at point C, and (d) the volume at point C. (e) Now consider the processes A → B, B → C, and C → A. Find Q, A, and ΔU_{int} for each of the processes. (g) For the whole cycle A → B → C → A, find Q, A, and ΔU_{int}.

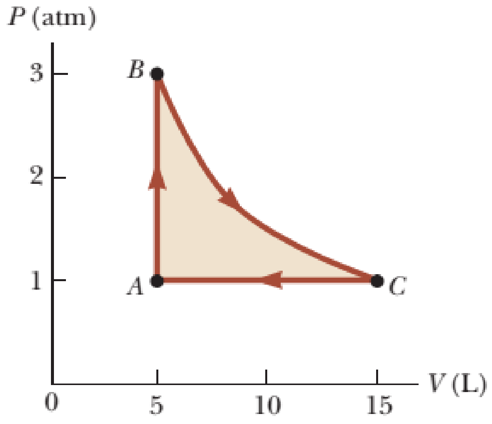


Figure 8.2.

Topic 9. Thermodynamic cycles. The second law of thermodynamics.

Example 9.1. Thermal efficiency of the cycle

An ideal two-atomic gas containing the amount of substance $\upsilon = 1$ mol is under a pressure of $p_1 = 250$ kPa and occupies a volume of $V_1 = 10$ liters. First, the gas is isochorically heated to a temperature of $T_2 = 400$ K. Then it is isothermally expanded back to the initial pressure. After that the gas is returned to the initial state by isobaric compression. Determine the thermal efficiency of the cycle.

Solution:

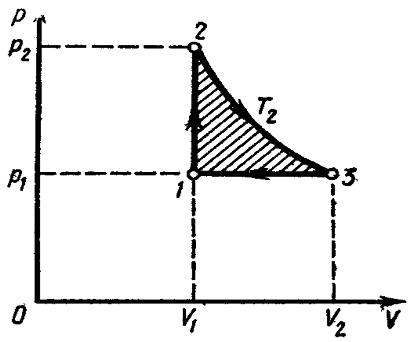


Figure 9.1.

First, build the pV-diagram of the thermodynamic cycle. It consists of an isochore, an isotherm, and an isobar (see Figure 9.1, the characteristic points of the cycle are denoted by 1, 2, 3).

The thermal efficiency of any cycle is determined by the expression

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1},$$

where Q_1 is the energy absorbed by heat from a hot source; Q_2 is the energy expelled by heat to a lower-temperature sink during the cycle. The difference $Q_1 - Q_2$ is the useful work A, performed by the gas per cycle. This work on the pV-diagram (Figure 9.1) is represented by the area of the cycle (shaded).

The working substance (gas) absorbs the energy by heat at two stages: $Q_{1\text{-}2}$ at the stage 1-2 (isochoric process) and $Q_{2\text{-}3}$ at the stage 2-3 (isothermal process). Then,

$Q_1 = Q_{1\text{-}2} + Q_{2\text{-}3}$.

The energy absorbed by the gas by heat in the isochoric process is

$$Q_{1-2} = \upsilon C_v (T_2 - T_1),$$

where C_v is the molar heat capacity of the gas at constant volume; υ is the amount of substance. We find the temperature T_1 of the initial state using the equation of state for the ideal gas:

$$T_1 = \frac{p_1 V_1}{\upsilon R}.$$

Substituting the numerical values, we obtain

$$T_1 = \frac{250 \cdot 10^3 \cdot 10^{-3}}{1 \cdot 8.31} = 300\,(\text{K}).$$

According to the first law of thermodynamics, the energy absorbed by the gas by heat in the isothermal process is equal to the work performed by the gas:

$$Q_{2-3} = A_{2-3} = \upsilon R T_2 \ln\frac{V_2}{V_1},$$

where V_2 is the volume occupied by the gas at the temperature T_2 and pressure p_1 (point 3 on the pV-diagram).

At the stage 3-1 (isobaric process), the gas expells energy by heat Q_2 to a cold sink,

$$Q_2 = Q_{3-1} = \upsilon C_p \left(T_2 - T_1\right),$$

where C_p is the molar heat capacity at constant pressure.

Substituting the obtained expressions for Q_1 and Q_2 we can find the thermal efficiency of the cycle:

$$\eta = 1 - \frac{\upsilon C_p \left(T_2 - T_1\right)}{\upsilon C_v \left(T_2 - T_1\right) + \upsilon R T_2 \ln\dfrac{V_2}{V_1}}.$$

According to the Gay-Lussac law, for the points 1 and 2 laying on the same isobar on the pV-diagram $\frac{V_2}{V_1} = \frac{T_2}{T_1}$. The molar heat capacities can be expressed in terms of the number of degrees of freedom of the molecule ($C_V = \frac{i}{2}R; C_P = \frac{i+2}{2}R$). Then, we obtain

$$\eta = 1 - \frac{(i+2)(T_2 - T_1)}{i(T_2 - T_1) + 2T_2 \ln\frac{T_2}{T_1}}.$$

Substituting the numerical values of i, T_1, T_2 and R, we obtain

$$\eta = 1 - \frac{(5+2)(400-300)}{5(400-300) + 2\cdot 400 \ln\frac{400}{300}} = 0.041 = 41\%.$$

Example 9.2. Carnot cycle

A heat engine is operating according to the inverted Carnot cycle. Its heater has a temperature of t_1 = 200 °C. Determine the temperature T_2 of the cooler, if the work done by the engine is A = 0.4 J, while the energy received by heat from the heater is Q_1 = 1 J.

Solution:

We can find the temperature of the cooler using the expression for the thermal efficiency of the Carnot heat engine

$$\eta = \frac{T_1 - T_2}{T_1}; \qquad T_2 = T_1(1-\eta).$$

On the other hand, thermal efficiency of the heat engine can be expressed as the ratio of the amount of heat that is converted into useful mechanical work A to the amount of heat Q_1, which is received by the working body of the heat engine from the environment (from the heater), i.e.,

$$\eta = \frac{A}{Q_1}.$$

Comparing these expressions, we can find

$$T_2 = T_1\left(1 - \frac{A}{Q}\right).$$

Considering that T_1 = 473 K, we obtain

T_2 = 284 K.

Example 9.3. Entropy

Find the change in the entropy ΔS in the result of heating of m = 100 g of water from the temperature t_1 = 0 °C to the temperature t_2 = 100 °C and subsequent transformation of water into vapor of the same temperature.

Solution:

Let us find separately the change in the entropy $\Delta S'$ when water is heated and the $\Delta S''$ when it is transformed into vapor. The total change in the entropy is the sum of the $\Delta S'$ and $\Delta S''$.

As we know, the change in the entropy is expressed by the formula

$$\Delta S = \int_i^f \frac{\delta Q}{T}.$$

Transfer of heat during heating of mass m of water equals

$\delta Q = mcdT,$

c is the specific heat of water, dT is the infinitesimal change in its temperature. Substituting the expression for δQ, we obtain formula for the change in entropy when water is heated:

$$\Delta S' = \int_{T_1}^{T_2} \frac{cmdT}{T} = cm\ln\left(\frac{T_2}{T_1}\right).$$

After the calculations, we obtain $\Delta S'$ = 132 J/K.

During transformation of water into vapor at the constant temperature T, the change in entropy can be found as

$$\Delta S'' = \int_i^f \frac{dQ}{T} = \frac{1}{T}\int_i^f dQ = \frac{Q}{T},$$

where Q is the amount of heat transferred during the transformation of heated water into vapor of the same temperature. Heat transferred to a substance of mass m during a phase change from liquid to gas is:

$$Q = rm,$$

where r is the latent heat of vaporization. So, we obtain

$$\Delta S'' = \frac{rm}{T}.$$

After the calculations, we obtain $\Delta S''$ = 605 J/K.

The total change in the entropy during heating of water and its subsequent transformation into vapor is

$\Delta S = \Delta S' + \Delta S''$ = 737 J/K.

Example 9.4. Entropy

Find the change in entropy ΔS during isothermal expansion of m = 10 g of oxygen from the volume V_1 = 25 liters to the volume V_2 = 100 liters.

Solution:

Since the process is isothermal, $T = const$, the change in entropy can be found as

$$\Delta S = \int_i^f \frac{dQ}{T} = \frac{1}{T}\int_i^f dQ = \frac{Q}{T}.$$

The amount of heat Q obtained by the gas can be found from the first law of thermodynamics: $Q = \Delta U + A$. For the isothermal process $\Delta U = 0$, therefore,

$Q = A,$

and work A for this process is determined by the formula

$$A = \frac{m}{M} RT \ln \frac{V_2}{V_1}.$$

Then,

$$\Delta S = \frac{m}{M} R \ln \frac{V_2}{V_1}.$$

Substituting the numerical values, we obtain

$$\Delta S = \frac{10 \cdot 10^{-3}}{32 \cdot 10^{-3}} 8.31 \cdot \ln \frac{100 \cdot 10^{-3}}{25 \cdot 10^{-3}} = 3.60 \text{ J/K}.$$

Example 9.5.

Two moles of an ideal gas are first isochorically cooled and then isobarically expanded so that the temperatre of the gas in the final state returns to its initial value. Find the change in the entropy of the gas if the pressure during the described process has changed n times.

Solution:

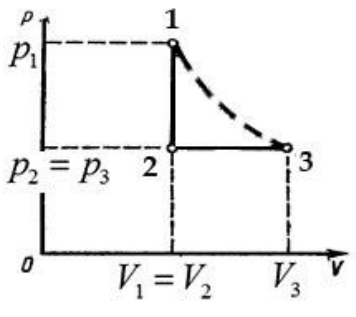


Figure 9.2.

First, build the pV-diagram of the process. It consists of an isochore 1-2 and an isobar 2-3 (see Figure 9.2).

Let us find separately the change in the entropy $\Delta S_{1\text{-}2}$ in the process 1-2 and the $\Delta S_{2\text{-}3}$ in the process 2-3. Then the total change in the entropy is

$$\Delta S = \Delta S_{1\text{-}2} + \Delta S_{2\text{-}3}.$$

The infinitesimal change in the entropy is expressed by the formula

$$dS = \frac{\delta Q}{T} = \frac{\upsilon C dT}{T},$$

where υ is the amount of substance, C is molar heat capacity of the gas; dT is the change in the gas temperature.

During the isochoric process 1-2, the amount of heat absorbed by the gas equals to $\delta Q_{1-2} = \upsilon C_V dT$, where $C_V = \frac{i}{2}R$ is the molar heat capacity at constant volume. Then, $dS_{1-2} = \frac{\upsilon C_V dT}{T}$, and

$$\Delta S_{1-2} = \upsilon C_V \ln\left(\frac{T_2}{T_1}\right).$$

According to the Charles's law for the isochoric process, $\frac{T_2}{T_1} = \frac{p_2}{p_1} = \frac{1}{n}$. Then,

$$\Delta S_{1-2} = -\upsilon C_V \ln n.$$

During the isobaric process 2-3, the amount of heat absorbed by the gas equals to $\delta Q_{2-3} = \upsilon C_p dT$, where $C_p = \frac{i+2}{2}R$ is the molar heat capacity at constant pressure. Then, $dS_{2-3} = \frac{\upsilon C_p dT}{T}$, and

$$\Delta S_{2-3} = \upsilon C_p \ln\left(\frac{T_3}{T_2}\right).$$

However, we know that the final and the initial temperatures of the gas are equal $T_3 = T_1$. Then, $\frac{T_3}{T_2} = \frac{T_1}{T_2} = n$, and

$$\Delta S_{2-3} = \upsilon C_p \ln n.$$

Now we can find the total change in the entropy:

$$\Delta S = \upsilon C_p \ln n - \upsilon C_V \ln n = \upsilon \ln n (C_p - C_V) = \upsilon R \ln n .$$

Problems

1. A gas is taken through the cyclic process described in Figure 9.3. (a) Find the net energy transferred to the system by heat during one complete cycle. (b) If the cycle is reversed—that is, the process follows the path ACBA—what is the net energy input per cycle by heat?

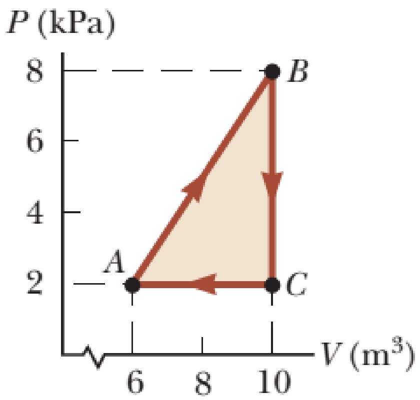


Figure 9.3

2. A sample of an ideal gas goes through the process shown in Figure 9.4. From A to B, the process is adiabatic; from B to C, it is isobaric with 100 kJ of energy entering the system by heat; from C to D, the process is isothermal; and from D to A, it is isobaric with 150 kJ of energy leaving the system by heat. Determine the difference in internal energy between B and A.

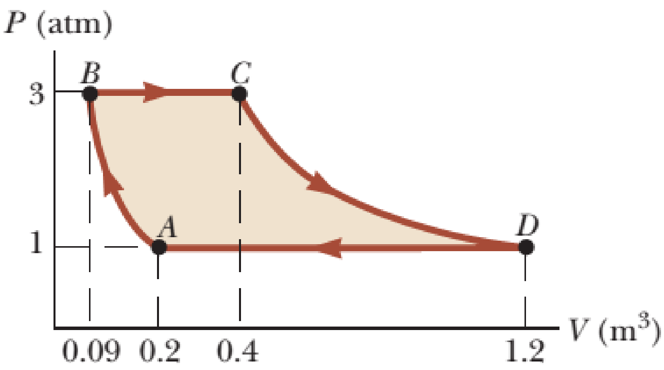


Figure 9.4.

3. An ideal gas initially at P_i, V_i, and T_i is taken through a cycle as shown in Figure 9.5. (a) Find the net work done on the gas per cycle for 1.00 mol of gas initially at 0°C. (b) What is the net energy added by heat to the gas per cycle?

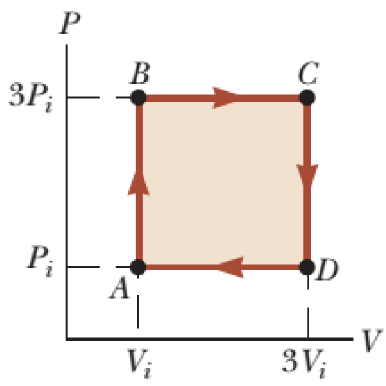


Figure 9.5.

4. An engine absorbs 1.70 kJ from a hot reservoir at 277°C and expels 1.20 kJ to a cold reservoir at 27°C in each cycle. (a) What is the engine's efficiency? (b) How much work is done by the engine in each cycle? (c) What is the power output of the engine if each cycle lasts 0.300 s?
5. The work done by an engine equals one-fourt h t he energ y it absorbs from a reservoir. (a) What is its thermal efficiency? (b) What fraction of the energy absorbed is expelled to the cold reservoir?
6. A gun is a heat engine. In particular, it is an internal combustion piston engine that does not operate in a cycle, but comes apart during its adiabatic expansion process. A certain gun consists of 1.80 kg of iron. It fires one 2.40-g bullet at 320 m/s with an energy efficiency of 1.10%. Assume the body of the gun absorbs all the energy exhaust—the other 98.9%—and increases uniformly in temperature for a short time interval before it loses any energy by heat into the environment. Find its temperature increase.
7. A Carnot engine has a power output of 150 kW. The engine operates between two reservoirs at 20.0°C and 500°C. (a) How much energy enters the engine by heat per hour? (b) How much energy is exhausted by heat per hour?
8. An ideal gas is taken through a Carnot cycle. The isothermal expansion occurs at 250°C, and the isothermal compression takes place at 50.0°C. The gas takes in $1.20 \cdot 10^3$ J of energy from the hot reservoir during the isothermal expansion. Find (a) the energy expelled to the cold reservoir in each cycle and (b) the net work done by the gas in each cycle.
9. An ice tray contains 500 g of liquid water at 0°C. Calculate the change in entropy of the water as it freezes slowly and completely at 0°C.
10. A Styrofoam cup holding 125 g of hot water at 100°C cools to room temperature, 20.0°C. What is the change in entropy of the room? Neglect the specific heat of the cup and any change in temperature of the room.
11. A power plant, having a Carnot efficiency, produces 1.00 GW of electrical power from turbines that take in steam at 500 K and reject water at 300 K into a flowing

river. The water downstream is 6.00 K warmer due to the output of the power plant. Determine the flow rate of the river.

12. A 1.00-mol sample of an ideal monatomic gas is taken through the cycle shown in Figure 9.6. The process A → Bis a reversible isothermal expansion. Calculate (a) the net work done by the gas, (b) the energy added to the gas by heat, (c) the energy exhausted from the gas by heat, and (d) the efficiency of the cycle. (e) Explain how the efficiency compares with that of a Carnot engine operating between the same temperature extremes.

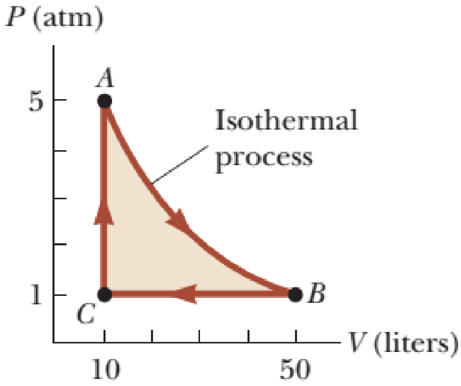


Figure 9.6.

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